

A

1/ We have been occupied with the study of an important kind of determination, called 'proper predication'. In it we have discriminated, among other things, a concrete thing determined and an abstract determinant determining it. Abstract things are structured in a system through the 'relation' of determinability. And we have had formulated what could be termed 'the law of determinateness', according to which for every case of proper predication, there is an absolutely determinate determinant of the abstract thing involved in it, which also characterises the concrete thing characterised by the said abstract thing.

2/ Now we naturally come to the question of the mode of existence of the abstract things involved in proper predications.

3/ Before proceeding, and with the intention of gaining a more general view, let us discriminate further elements involved in the examined kind of determination. And let me firstly introduce the notion of an instance of an abstract thing. This can be done, as neutrally as possible, through the following definition:

in case that a is P, an instance of P-ness is the P-ness of a.

But now note an ambiguity in the expression "the

P-ness of a". It may be used, on the first hand, to specify a certain kind of P-ness, by giving an object exemplifying it, by giving an example. (Such specification may be expedient and even, in some cases, necessary: expedient, if, say, the kind specified is nameless and we want a handy way of identification of the kind meant; necessary, if both the kind and the means of defining it as a determinate of P-ness are nameless and no adequate paraphrases are available). In such a case, of course, the P-ness of a may be shared by other particulars as well---even if it specifies, under the given conditions, an absolutely determinate determinant. But, on the second hand, the expression may be used in such a way as to be possibly applied even where P-ness itself is an absolutely determinate determinant. In a case like this, it obviously cannot be explained as above; for simply there are no kinds of absolutely determinate abstract things (I mean sub-kinds of them). In the present application it has to mean something which cannot possibly be shared by anything besides a; the expression "the P-ness of a" characterises also b* expresses an impossibility.

It is in this second sense that I am using the expression 'the P-ness of a' in the above explication of 'instance of P-ness'.

As the above ambiguity makes it clear, we must carefully distinguish between a possibly shared absolutely determinate determinant and an instance of an abstract thing which cannot be shared, as it belongs to just one concrete thing (exemplifying the abstract thing), by reference to which it is defined.

4/ An instance of an abstract thing is inseparable from the concrete thing to which it is attached. By 'inseparable' I mean that it cannot be separated from it, in reality or in thought. In other words, it is not possible for a given instance to 'exist' (in whatever sense it exists) in any other way, save by being specially attached to the concrete thing, in reference to which it is defined. And it is not possible for the instance in question to be conceived as existing otherwise. Let us call the special mode of existence of instances (which is at the same time a mode of connection to their subjects), 'being in'. Instances are only in concrete things, and each instance is in one and only one concrete thing.

Note that this appellation is introduced above to refer to just one aspect of the mode of existence of instances, namely to their inseparability from their corresponding concrete things. Of course, this one aspect may prove to be a fundamental one---or even the only one in case that the ontological status of instances as entities proves to be weak enough. Anyway, generalising, we use the appellation to mean the mode of existence of instances in general, whether it turns finally to be restricted to just that mentioned aspect or no.

5/ Returning now to the question of 2, we can see it as a special case of a more general question which we can, perhaps, formulate as follows:

What are the entities involved(as elements or components)in proper predication? And how are they,i.e. what is their mode of existence?

(I leave it open for the time being whether the above questions really constitute one question, or two closely connected questions , or two more or less unconnected questions).

6/ We have distinguished in respect of a proper predication the following items :

- i/ the fact constituted by it
- ii/ the determined thing
- iii/ the abstract determinant
- iv/ the tie connecting(holding together) (ii) and (iii) when the fact (i) is actually constituted.

We have also distinguished in proper predication the following aspects(corresponding to the above items; the difference is a difference of formulation):

- i'/ the determination as a whole
- ii'/ that which is determined in the determination
- iii'/ that which determines in the determination
- iv'/ the determination of the determined by the determinant(the determination in a narrow sense considered in abstracto from both the determined and the determinant)---which has both an active

and passive sense corresponding to whether it is viewed from the point of view of the determinant or of the determined.

7/ But now we have introduced instances. What is their connection with the above distinctions? Let us, provisionally, acknowledge instances as a fifth element involved in proper predication. We then can formulate the connection of it with the above 4 distinctions in the following way:

An instance is in (or inheres in) a concrete thing if and only if the abstract thing of which the instance is an instance, determines the said concrete thing.

8/ To tie the above 'divisions' more closely: We may, I am inclined to think, discriminate still another sense of 'determination'. Perhaps the best way of explaining it, is by using the analogy of activity (action and passion). (Of course, the analogy of this analogue to proper predication--and determination in general--is not complete, extending to some of their aspects; but, I conceive, the analogy holds good in some relevant and important respects).

When a thing a exercises some activity upon a thing b, a is the active and b the passive thing in respect of that activity. Now, the activity produces some effect in b, say a new quality or state of it, which serves, so to speak, as the mark left on b of a's activity upon it. This effect is the result of the activity (at least in a certain sense, for in another, that result is the change effected in b).

It follows, that in an activity we have to distinguish, of course, the active and passive things, and further to distinguish two different (though, naturally connected) senses of 'activity': the activity as a whole including the active and passive things as well as the action of the former upon the latter; and the activity as that action considered in abstracto from the acting and acted upon things, i.e. the activity as the 'relation' or connection holding between these two things when the one acts upon the other. Finally we must discriminate the result of the action, i.e.

that aspect of the passive thing, which is produced through the action and which serves, as it were, as a natural sign of it.

Returning now to determinations, we see that the result of the action, corresponds to a third sense of 'determination', namely to the determination as the mark, so to speak, left upon the determined thing as the 'result' of the 'action' of determination.

(The inverted commas of " 'result'" and " 'action'" above help to bring into the mind the circumstance that we are simply using here clarifying analogues).

Perhaps it is now clear that we have to add to the second list of 6, the above mentioned sense of 'determination' as a fifth item.

So that, rewriting the list we obtain:

i) determination as a whole including the determined thing and the determinant, as well as the 'activity' of the second upon the first, when the second determines the former.

ii') determined thing

iii') abstract determinant

iv') determination as the 'activity' mentioned in (i') above, (conceived in abstracto from the acting and acted-upon things); such that when it is 'exercised', determinations in sense (i') are constituted

v') determination as the 'result' of the said 'activity' upon the determined, when that 'activity' is really exercised upon it by the determinant.

9/ Falling back upon the second analogue of determination utilised in Essay II, namely that of relations, we can rewrite the above list as follows:

i') determination as a whole including the determined thing and the determinant, as well as ~~and~~ the 'relation' holding between them when the second determines the former

ii') determined thing

iii') abstract determinant

iv'/ determination as the 'relation' mentioned in (i') above (conceived in abstracto from the things it 'relates'), such that when it holds, determination in sense (i') to be constituted.

v'/ determination as the relative quality of the determined, when it really stands in the 'relation' (ii') to an abstract determinant, i.e., when it is in fact determined by that determinant.

(The inverted commas of "'relation'", help to bring into mind, that, as I have argued in Essay II, determination (in sense (iv')) is not an ordinary, proper relation, because proper relations are just a kind of determinants entering into the pseudorelation of determination together with concrete things--- and surely determination is not a kind of determinant.)

We may find it convenient to keep to the following terminology: relations in themselves and ties are connectors (that is, objects connecting or working connections---the ontological grounds of connections); in this way, to relations as holding among (concrete) things there correspond the tie-type connections.).

10/ The lists of 8 and 9 are now in complete agreement with the following list (which is an extention of the first list of 6).

i/ fact

ii/ determined thing

III/ abstract determinant

iv/ tie (of determined to determinant)

v/ instance

Hence, this seems to be a complete catalogue of the entities which, *prima facie*, we have to acknowledge as being involved in some way or other in proper predications. Still, it is true, we must further, for completeness, include in the above list the following items:

vi/ tie of instance to determinant

vii/ tie of instance to determined.

II/ I use the word 'tie' in a way to include only connections (conceived in abstracto from the connected items, as that

which makes them, when applied to them, to constitute the connections as wholes) of items belonging to different categories. Hence relations are not ties, since, being determinants, they connect concrete things. Besides, relations as determinants stand in need of a tie in order to be possible for them to 'relate' concrete things, so that, according to the above analysis, in a relational fact of the form 'aRb' we can distinguish the concrete things a and b, the abstract determinant R-ness and the tie which connects R-ness to a and b. We may, if we so choose, speak of "connectors" to include both relations and ties----and this is all right as far as terminological convenience extends. The substantial problem would then be, of course, whether this stipulatively common name signifies a nature common to both relations(as connectors of concrete things) and ties(as connectors of abstract and concrete things).

..... Anyway the ties (iv), (vi) and (vii) are clearly non-relational connectors.

B

I2/ But now, and before proceeding, an objection must be faced by all means. According to it, if we require a tie to tie the determined thing and the determinant together in order for the corresponding fact to be produced (or, better, constituted), then we seem also to require a tie to tie the first to the said things. And clearly this is the beginning of a regressus ad infinitum. Hence the initial requirement is called into question, and must be qualified or abandoned.

II/ There is a controversy regarding the validity of such arguments (or the nature of the conclusions that they really prove) which exhibit the regressus ad infinitum form. It is often claimed that regressi ad infinitum are not necessarily vicious, though they may well be for other reasons than the infinity exhibited in them. But in such a case we have to seek for some general account of the viciousness of those among them which are vicious, and their difference from the others.

Let us then try initially towards this direction.

Suppose that a certain 'move' m_0 is proposed in respect to a certain problem, as the solution of it or as contributing substantially to its solution. The move in question may be the posing of an entity or a definition of the nature of something, or some explanation etc.

Now that move may require another move m_1 as condition of its validity; or it may be the case, on the other hand, that it entails a consequent move m_1 . So that it may be the nature of the

initial move to have a series of antecedent conditions or a series of consequents.

(Of course, strictly speaking, it is the content of the move rather than the move itself, which is conditioned (requires antecedents) and entails consequents. But then the moves may be said to stand in the 'relation' of condition and consequence secondarily, if their contents stand in these 'relations').

(By 'content of a move' I mean the proposed objective 'fact', whose representation, assertion and introduction into the discourse is made by the corresponding move.)

So, if the move is the posing of an entity, its content is the existence of that entity, or rather, that entity assumed and 'asserted' as existing. Similarly, in the case of a definition, the corresponding content is the nature stated in it, and in the case of an explanation of something's occurrence, it is the reason or cause (depending on the nature of the case) of that occurrence.).

13/ It might be urged that, as it appears, one cannot distinguish between consequences and conditions (at least necessary conditions) on purely and strictly logical, formal grounds. For if $p \rightarrow q$, q may be either a necessary condition of p or a consequence of it.

And indeed the distinction may not be possible to be made in purely formal terms. But once we have access to semantical (and in the last analysis ontological) considerations, it would seem that the distinction must be correct and important. For apparently A's being male is a condition of A's being a father, though equally plausibly, $p \vee q$ may be held to be a consequence of p , or the fact that the sum of the angles of a plane figure equals two right angles, is a consequence of the fact that it is a triangle.

The intuitive idea is, of course, that in order for a (logical, formal) entailment $p \rightarrow q$ to be a real consequence, ' q ' must be true, if true at all, by reason of the truth of ' p ', not simply because we can know (by various means) that given that ' p ' is true, ' q ' must also be true----the requirement of the impossibility of the case in which ' q ' is true and ' p ' false, does not suffice by itself to give, or rather to constitute a real consequence, though it suffices to constitute a formal entailment out of a simple (material) implication.

But how can we articulate this intuitive idea? Not by relying on formal-logical considerations alone. We have to examine the ontological status, so to speak, of the items combined by the 'relation' of entailment, as well as of the elements involved therein.

Let us study the examples mentioned above:

i/ (A is a father) \rightarrow (A is male)

ii/ (A is a triangle) \rightarrow (A has angles equalling to two right angles).

In (i) maleness is a proper quality of men and fatherhood a relation holding among human beings. (We do not consider other animals). The quality may belong to a subject or not, yet its belonging is not due to the subject's standing to a certain relation (that of fathership) to other persons. That someone is male, is, evidently, not so in virtue or by reason of his being a father. On the contrary the possibility itself of something's becoming a father depends on its being male. Only male persons can be fathers; only in respect of a male person may the question legitimately arise of whether he is a father or not.

Quite the opposite, it would seem, holds with (ii). If a plane, rectilinear figure has its angles summing up to two right angles, then it has this property in virtue and by reason of its being a triangle. And further, it is not the case that the possibility of something's being a triangle rests on (is founded upon) its having angles equalling to two right angles. That possibility depends really on its being a plane, rectilinear figure--- and, indeed, the entailments:

III/ (A is a triangle) \rightarrow (A is rectilinear)
is of the condition-type (like (i)), not of the consequence-type.

In case that the contrast (i)-(iii) might be considered as a bit strained, because of the difference of the field from where the examples are taken, I should note the similar contrast (iii)-(ii), and, also, I should list the following examples of consequence-type entailments taken outside the field of a mathematical science.

iv/ (A chooses freely to do f) \rightarrow (A is responsible for doing f)

v/ (A is beautiful) \rightarrow (A is desirable (a proper object of desire))

vi/ (it is the case that p) \rightarrow (' p ' is true)

- vii/ (A promised to do f) \rightarrow (A ought to do f)
(doing f , in this case is not
an absolute, categorical duty)
viii/ (a is a sound) \rightarrow (a has a certain pitch)
ix/ (a is extended) \rightarrow (a is coloured)

15/ In the case of mutual entailment or logical equivalence(I do not mean material equivalence, needless to say) my point is perhaps even clearer, or at least, more readily seen.

For take, for example case (vi). Obviously, even though , from a purely formal point of view, a proposition is true if and only if the corresponding state of affairs(which is stated in it) obtains, yet nevertheless, a proposition is true in virtue or by reason of the obtaining of the fact and not vice versa. Similarly, mutatis mutandis, with (iv), too---if it is construed, as it apparently should, as a case of mutual entailment.

On the other hand a mutual entailment like the following:

x/ (A is a father) \rightleftarrows (A is a male parent)
is also 'directed'; still the 'if and only if' formula works, but A is a father by reason of his being a male person who has begotten another person, and not vice versa. However, the 'direction' is here of a different kind relating also to the priority of parenthood as the determinable of fathership.

16/ It is to be emphasised, that the above attempt at distinguishing two subtypes of entailment(though, it is true, from a broadly philosophical and not strictly formal, point of view) conforms only too well with our intuitive and natural, so to speak, aversion to identify logical equivalence(conceived as mutual entailment) with identity of meaning. A contrast between (x) and (vi) exhibit what I want to say. But I shall not pursue here this issue.

17/ Can we give a somehow general account of the difference between Cd-type(condition-type) and Cs-type(consequence-type) of entailment?

But let us note, firstly, that this difference amounts also to a difference in kind of the necessity attributed to the two types of entailment; and that this latter difference is quite

another thing from the difference between logical and natural necessity, as it apparently cuts through this distinction.

Having said this, I propose, hesitatingly, the following principle of division of entailments:

1 / Condition-entailments are entailments whose validity rests, is grounded on the structured system of determinants or abstract things (as structured by the determinable/determinate 'relation') in a way analogous to the following:

$$(a \text{ is } P) \longrightarrow (a \text{ is } Q)$$

where Q-ness is a determinable of P-ness or a differentia constitutiva of P-ness or of a determinable of P-ness.

2 / We then may 'define', provisionally, consequence-entailments as entailments which cannot be brought under the above scheme of deduction of their validity.

(We must note, firstly, that it remains open whether (2) 'defines' a homogeneous class of entailments, as it should do, according to the principle that a proper and correct division must not contain amiscellaneous category--to put it vividly.).

(We must not, secondly, that the above principle of division applies to logical entailments. In the case of natural entailments and natural necessity time seems to provide the means of distinguishing a condition of a given state of affairs from a consequence of that state. But time is only a partial guide: a lapse of time is neither sufficient nor necessary condition of the validity of a natural entailment. But still if it occurs in respect of a natural entailment, it determines its being a condition or consequence entailment, though the absence of such occurrence leaves the matter undetermined. It is in terms of explanation that we, perhaps, can give a general account of the distinction in view, in the case of natural entailments. For if p explains q according to the natural laws, then p is a condition(a natural one) of q. It is a consequence, if the explanation holds in the opposite direction.).

C

18/ Returning now to² 13?A move and, primarily, its content may require for their validity or obtaining (correspondingly) a series of conditions---or, on the other hand, may have a series of consequents attached to it.

Now the whole question turns on the form and degree, so to speak, of finitism, to which one is committed. For a strict finitist, I suppose, for whom infinity itself is a negative concept (corresponding to which there is no determinate nature) meaning only the absence of determination in quantity¹, and hence nothing infinite not only actually exists, but even is possible to exist, is infinite², an infinite series of either conditions or consequents invalidates the move to which these series are attached and can be proved to belong. According to this view, infinity itself is the vicious element in a regressus ad infinitum argument.

But if one is not prepared to accept finitism in such severe form, he may attenuate it by not construing "determinate and infinite" as a contradiction in terms, though he may still insist that nothing infinite actually exists or even is possible to exist, but insist so, on condition that that nonexistence or impossibility

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1. We shall find occasion to modify these explications subsequently.
 2. Operating here at bottom is what I should call "the metaphysical law of determinateness" according to which, in a rough formulation, existence and determination are intrinsically connected. The precise form of this connection is to be evolved later on. Yet, we have an example in the case of the absolute determination of concrete things.

is due to a reason less radical and fundamental than the indetermination allegedly included in the concept of infinity; as such another reason, it might be proposed that the existence of anything infinite would annul everything else, save parts of it (I am not saying that this particular proposal would be correct).

A substantially weaker type of finitism would result if we were to restrict ourselves to consider only some kinds of infinity as inadmissible and illegitimate. From the point of view of such a form of finitism the question regarding the invalidity of a regressus ad infinitum argument is more complicated; for now its invalidity depends on the kind of infinity which can be proved to belong to the move upon which as base the regressus has been constructed, and not to infinity as such. In other words, the question now is not so much whether anything infinite can exist, but what kind of 'things' are susceptible of actual or potential infinity (i.e. are or can be infinite). A specific finitism of this kind (of this weak type) would be the one according to which only an infinite series of conditions is really vicious, whereas an infinity of consequents has nothing wrong with it.

19/ But a complication lurking here must be noted and unravelled. What is possible can be (can be the case), i.e. can exist actually---or, using the expressions 'it is possible that p' and 'p exists (obtains)' potentially; equivalently, we may say that what exists potentially can exist actually, and this is so, according to the nature of potential existence itself (and of possibility).

But now, we wish to hold, it is possible to keep dividing a given segment ad infinitum. But is it correct to say that an infinite division of it exists potentially and hence that it can be actually accomplished?

20/ It might wonder: "Why not?" But let us take the usual example of a rectilinear segment. This can be divided into two equal (spatial) parts; and when this is done (in reality or in thought), each of the two resulting segments can be further divided in its turn into two equal parts---and so on. Let us call 'a' the initial segment, 'a₁', 'a₂' its two parts resulting upon the first division, 'a₁₁', 'a₁₂', 'a₂₁' and 'a₂₂' the four parts resulting after the second division has taken place, and so on.

Now consider the segment a_{n_1, n_2, \dots, n_k} (where $n_i = 1$ or 2 for all values of i from 1 to k). It is one of the segments resulting from the k^{th} division of a . But what is the precise sense of 'resulting' in the above written sentence? In what sense does a_{n_1, n_2, \dots, n_k} results from the k^{th} division of a ?

The strongest possible answer to this question, apparently is to hold that through the division the sub-segment is constructed out of the initial segment, that the division is the reason of the existence of the said sub-segment, (or even the cause of its existence, if we have to do with material lines and segments), -- or finally, that the sub-segment exists in virtue of the (occurrence of the) division^I.

According to this thesis, it is, of course, the actual existence of the sub-segment that is 'caused' by the division; the sub-segment is constructed (that is, is made to exist actually) much as the statue is formed out of the bronze.

I. To people protesting that I use 'cause' with 'construct' and 'cause' with 'reason' in one and the same breath, I retort:

a/ Indeed there are differences between constructing and causing. In the first place construction is of items, cause is of the existence or occurrence or taking place of items. And further, in the second place, though every case of constructing seems to involve also a case of causing, the opposite does not seem to hold. For (to isolate some of the differences):

i/ we construct things but not events; constructing is causing the existence of a thing, not the occurrence of an event.
ii/ 'constructing' comports the immediacy of the acting cause, its proximity to the result, which is not necessarily so in cases of causing in general.

iii/ constructing usually (or, perhaps, even normally) involves intentions and (in most cases conscious) activity orientated towards an end---or, at least, it connotes the working for the production of something which, in virtue of its relative complexity and sophistication, could not be brought to exist accidentally or as a result wholly of chance influences. In the case of causing this is obviously indifferent.

But granted that there are such differences between constructing and causing, I trust that I have not erred in respect of those among them which are philosophically significant. For example (i) is rather irrelevant here, as we are concerned with thing-like entities (segments). The initially diagnosed difference is also taken care of by the formulation. (ii) is the less philosophically important, the more it is a question of degree of proximate ness. And the same remark applies partly to (iii), too. Besides, the nature of the examined example makes (iii) irrelevant, in so far as it involves to some extent the intentionality of constructing.

b/ I do not use 'reason' and 'cause' in opposition to each other---as for example the terms are usually used in respect of the explanations of human activity and behaviour. As I use them, the cause (in the modern scientific use) is the physical reason of a physical obtaining (existence or occurrence of something physical) or

21/ To such a strong and ontological view, it seems to be opposed a weak and gnosiological view of 'resulting' as here used. According to this second view, the sub-segment exists actually from the beginning, so to speak, as a part of the initially given segment. The only thing that the division does is to provide us with a way of discovering it, apprehending its, not only objective (non-subjective), but even independent of the division, actual existence.

But the seeming opposition of the above sketched views is rather deceptive. For if we take literally the principle implicit in the second thesis, we might by applying it to the bronze-statue case, conclude that the statue actually exists in the bronze even before the beginning of any process of forming it, and that such processes only give us a way of apprehending its actual existence which is independent of them! But then the point of the distinction between actuality and potentiality is lost. Clearly the statue is what results when such-and-such operations (the processes of forming) are exercised upon the bronze---and, correspondingly, the subsegment (qua part of the initial segment---that is, actual part) is what results when such-and-such operations (a series of divisions) are exercised upon the initial segment. For the statue is constituted (and 'defined') by a certain form actually drawn, and, in an analogous way, the subsegment is constituted (and defined) by certain boundaries (end-points) actually marked (cut, so to speak)---though marked either in thought (and then we have the subsegment existing in thought) or in reality, and if in reality, either in ideal reality (mathematical lines) or in physical reality (material lines).).

(Note , by the way, that, strictly speaking, a formula like ' a_{n_1, n_2, \dots, n_k} ' does not give us the subsegment even in thought ---it only gives the way of conceiving the subsegment through the following of the series of divisions signalled by the formula, and therefore it only gives a way of constructing it in thought or in reality.).

22/ Returning to 20. We can say: the initial segment

of an obtaining in general, in so far as it is physical or has a physical basis, component or aspect.

has potentially as part the said subsegment---and has actually it when the corresponding divisions are executed. We also can say that the subsegment exists potentially in the segment(as potential part of it)before the taking place of the divisions, and actually after their occurrence? Finally we can say that the divisions involved are possible, and therefore can occur.

Therefore, we see, (a) and (b) of § 19 are equivalent or, rather, necessarily connected, since, strictly speaking, it is the division which is possible and its result which exists potentially.

23/ But now what precisely do we mean when we wish to say that the division can continue ad infinitum? In the sense of 'possible' so far understood, possibility entails possible actuality. But then do we wish to hold that an infinite division of a segment may actually occur, as an infinite division? That is, do we really wish to hold that a division can indeed(in the so far accepted as standard sense) continue ad infinitum?

Of course for strict finitism(as the one in § 18) the solution to this problem is rather simple. Continuation ad infinitum of the division means indefinite continuation of it, that is, it means the proceeding from subdivision to subdivision (or rather the possibility of such proceeding) without involving any reference to the number of steps taken(or to be taken) in this procedure. Infinite in this connection, means indefinite, indeterminate. Hence, according to this view, the possibility of an infinite of an infinite division so called, is nothing more than the possibility of dividing any given segment into two equal parts. For in speaking of the possibility of an infinite division(an infinite series of divisions), we abstract from the element of finiteness(i.e. determinateness) involved in each and all actual or really(in the proper sense) possible(permit me the expression) series of divisions, and we choose to concentrate on the principle of division itself(common to all such actual or possible series), according to which any given segment can be divided into two equal parts. So that the correct abstraction would be the conception of the common nature of all actual or just possible series of divisions as comprising firstly the above stated principle and secondly the requirement of a definite(that is finite) number of its application in any given case. By abolishing the second member of this conjunction, we then suppose to form the

pseudo-conception of a nature to which the second requirement is not essential (is not contained in its definition), and which could be posed as the genus under which the conjunctive nature above defined (more correctly: the nature answering to the double requirements above given) would be a simple species---mistaking in this way the possibility of any determination for the possibility of indeterminateness.

And indeed, this last formulation, seems to pinpoint the error committed in all 'infinitisms'----at least according to the view advocated here for the sake of argument, but which, I must confess, I am much inclined to adopt. For the so called (very misleadingly indeed) 'possibility of dividing ad infinitum' is nothing more than the possibility of proceeding (pushing) the division one more step ahead of (beyond) the point already reached, at each instance; it is the normal possibility of dividing again from the stage arrived at (whatever this stage is), and not a queer possibility of accomplishing a non-finite series of divisions.

24/ According to the stated view then, we can perhaps succinctly formulate the error committed by infinitism (to speak collectively) as follows:

i/ From 'it is possible to continue the operation of division for K times' it follows 'a k-fold division of a given segment can be accomplished'.

whereas:

ii/ From 'it is possible to continue the operation of division ad infinitum' it does not follow 'an infinite division of a given segment can be accomplished'.

Or, in other words, in order to connect with 219:

i'/ 'it is possible to continue the division of a segment a for k times' entails ' a_{n_1, n_2, \dots, n_k} exists potentially (as a (potential) part of a)'

whereas:

ii'/ 'it is possible to continue the division of a segment a ad infinitum' does not entail ' $a_{n_1, n_2, \dots, n_k, \dots}$ exists potentially'. Indeed $a_{n_1, n_2, \dots, n_k, \dots}$ does not exist even potentially; it is a complete nonentity. (Or, in the contemporary style:

the expression ' $a_{n_1, n_2, \dots, n_k, \dots}$ ' violates the formation rules of the language to which it belongs. (True, we mean formation rules for expressions which are not sentences---but if this should be objected, we can be transposed to something like: ' $a_{n_1, n_2, \dots, n_k, \dots}$ is a part of a' , and here we do have a collocation of signs purporting to be a meaningful sentence.).)

Now, one way of explaining why (i') and (ii'), as opposed to (i) and (ii), hold, is to say that 'it is possible to continue the division of a segment a for k times' entails 'it is possible for a to be divided k times', whereas 'it is possible to continue the division of a segment a ad infinitum' does not entail, 'it is possible for a to be infinitely divided'. Indeed it is impossible for a to be infinitely divided, if it is correct that 'infinitely' means here indefinitely: a segment cannot be divided into a number of parts, without being divided into a certain, definite number of parts.

To say the same thing in a still somehow different way: Let us consider:

a/ we can divide a , k times

b/ we can divide a ad infinitum.

The propositions (a) and (b) are of very different forms in spite of their external and superficial similarity. For the possibility involved in the 'can' of (a) is directly the normal modality, whereas the 'can' of (b) does not express this normal modality (I say 'directly', because, after all, it does express it indirectly in so far as it has sense).

Or, better, viewing it from the other end, the operand upon which the normal modal operator of possibility operates in (a) is the k -fold division, whereas in (b), the corresponding operand is not an infinite (indefinite) division, but any finite division.

25/ One may be worried from the fear of a lurking confusion in the above discussion between human and logical impossibilities. But I do not think that the impossibility of an infinite division of a segment has anything to do with human powers and incapacities. It is rather a question of ontological impossibilities: things are determinate---an indeterminate (in any way) thing is an impossibility. A special case of this principle of determinateness is the impossibility of the existence of anything indeterminate in respect of quantity (both extensive and intensive quantity), and so, of anything infinite.)

D

26/ But still, one might insist, even granted that the above offered analysis of infinity in respect of (continuing) divisions of segments is valid, there are other cases where we really wish to hold that the concept of infinity is a positive one denoting a real nature (and not the negation or privation of a certain definite (though perhaps very general and abstract) nature). For we can indeed, can we not?, form the apparently legitimate conception of, e.g. an inexhaustible source or of an infinite power. And what is impossible in them? It is mainly from such examples, I conceive, that we are inclined to consider finite and infinite as contrary & positive natures and not to consider the second as simply the privation or negation of the first. For contrariety implies positive existence of the contraries. And so, in such a view, both finite and infinite quantity is determinate quantity, thou differing in kind perhaps (but not in genus).

27/ So let us firstly examine inexhaustibility. Inexhaustible is that which cannot be exhausted by subtraction. And here, it seems at first sight as if the question was not whether the concept of inexhaustibility itself is a legitimate one, corresponding to some definite (however abstract) nature, but on the contrary, whether there is anything which can be inexhaustible----'can be' not in the sense of abstract possibility of an exemplification of inexhaustibility, but in the sense of there being a thing with such a nature as to permit its being susceptible of inexhaustibility.

But is this first appearance correct? Or how far is it correct? We have stated above that inexhaustible is that which cannot be exhausted by subtraction. Therefore this impossibility of exhaustion is just the other face of a required possibility of continual subtraction (subtracting each time a definite, finite quantity). And this latter corresponds to the 'possibility' of continual division which we have previously examined.

But a quite important difference would seem to emerge immediately here---a difference proving that the present case is much more difficult and intricate than the beforegoing one. For there, we could comfortably ascribe a definite sense to the possibility involved by explaining it as the possibility of making once more the step which has been already a definite number of times (whatever this definite number is)---and not as the possibility of attaining to the misleadingly supposed objective of accomplishing a process consisting in an infinite number of steps (and this, irrespective of questions regarding human capacities). But now, here, we also have apparently to acquiesce, even if we cannot do it equally comfortably as in the former case, in saying that the possibility involved is the possibility of subtracting once more after any definite number of subtractions, whatever this definite number is---even if the possibility of the subtraction being actually made an infinite number of times is not so involved. But here, in saying even the former, we clearly hold by implication, so it would seem, that a quantity can be really and actually not-finite in such a case, since if it was finite, there would exist a certain, definite number of definite subtractions (subtractions of definite quantities) which would exhaust the initial quantity if applied to it. Hence we seem to recognise in such a case the legitimacy of some form of a positive concept of infinity. Of course there may still be the case that in our World nothing infinite exists; but this is relevant to the present issue, in which the repugnancy of a positive concept of infinity itself to reason and reality is at stake.

28/ To say the same thing in different words. We have explicated a sense of infinity, according to which the possibility of something infinite is the possibility of an operation's occurring any (finite) number of times. This is clearly a sense ontologically non-committal, in that, on the one hand, does not necessarily entail anything

about the subject upon which the operation is effected or exercised^I, and on the other hand does not even require the accomplishment of an infinite number of successive applications of the operation. (Indeed, we have seen, such a pseudorequirement is the result of a misunderstanding of the 'nature' of infinity involved here, which, essentially, is nothing more than a certain 'openness' or availability for any determination).

But we have also seen that in some cases, this neutral and noncommittal sense of infinity as a purely privative concept pertaining primarily to operations rather than to concrete things or to ~~the~~ things, seems to entail grave consequences of a much more ontologically committed character. For it seems to demand an actual positive infinity in respect of the subject, of the thing (or, at any rate, of what is posed and considered as thing); which actual infinity of it, is to serve now as a source and base of the possibility of the never-stopping operations, and hence of the neutral kind of infinity in respect of the operations applied to the subject.

29/ We may put the result in figurative language by distinguishing two kinds, as it were, of infinity: one corresponding to the overdetermination of the inexhaustible, the other corresponding to the underdetermination of the indefinite. The second is the privation of any determination, the first the source of any determination proceeding from it.

We see now that, as it seems, the real and crucial problem regarding infinity is not so much posed either through infinite divisibility of extention, or by the infinite multiplicity of arithmetical determination (the infinite multiplicity of numbers; for, very roughly speaking, 'there are infinite numbers' means: any discrete quan-

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- I. In truth, it indeed entails that the subject is susceptible of that operation. And more than this, it entails that the subject is susceptible of reiterative application of the operation concerned---and even more, that it is susceptible of any number of successive applications of the operation. But in spite of all this, I only meant to say that nothing follows regarding the ontological finiteness or infinity of the subject---which is the proper issue here.

tity can be measured, however great). The main difficulty, so it appears now, relates to the concept of inexhaustibility.

30/ But of course we can see things from the other end too. And instead of saying that because the infinity of some operations (as this was previously explained) requires a corresponding positive infinity of things (or rather of their quantities) we have to admit as legitimate this concept of a positive kind of infinity, instead of this, we may hold that precisely the validity of such a requirement invalidates the corresponding series of operations; however faultless it might have appeared initially.

On this account (which of course sticks to the privative 'nature' of the concept of infinity) we have to say that simply it is not really possible to subtract from any possible quantity finite portions ad infinitum; for this would make the magnitude in question indefinite and indeterminate, and hence no quantity at all, only, at most, the matter of a (anyone) quantity-determination. A quantity is a determinate quantity surely---that is, a finite quantity I.

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- I. The above solution both calls for, and is susceptible of further elaboration in respect of its dialectical persuasiveness.

E

SIP. But before beginning to disentangle the various threads of the various examined issues and allocate them appropriately, let us face directly the central problem, asking for the nature of infinity---a problem which any metaphysical system has sooner or later to face.

Now finiteness and infinity pertain to quantity; and one preliminary step before confronting the central problem is to investigate in what sense do they pertain to it.

Let us then propose that finiteness and infinity are determinants^I determining quantity. And by this I mean something analogous with what we mean by saying that oddness and evenness are determinants determining number. But we must beware of a confusion here: oddness and evenness are not determinates of the determinable number, though, of course, odd-number and even-number are. Similarly with finiteness and infiniteness. Still, they do seem to be in one or another sense per se attributes of number. (Yet I shall recline from further complicating the issues here by introducing problems regarding the concept(s) of *κατέτο*). Anyway, they are οὐνέκι διαφοραί (proper differentia) of quantity, alongside with, say, discreteness and continuity.

I. I use 'determinant' here loosely, and, strictly speaking, improperly (not in the technical sense) to denote, possibly, even something privative or negative in nature---not only proper determinants which, of course, correspond to positive natures.

Now this last pair furnishes us with a contrasting background for our investigation. For quantities are indeed discrete or continuous; and both discreteness and continuity are positive natures, thought apparently contraries. Now is it in the same way the case that quantities are just, possibly, finite and infinite? That finiteness and infinity are both positive, though contrary, natures?

Let us examine what may be in an affirmative answer to the above questions. The first question to be asked is, I suppose, whether infinity is susceptible of more than one specific determinations (granted, of course, that it is something definite---in the way abstract things can be definite) in the way that finiteness is so susceptible, or no, in which case an infinite quantity is just one certain definite, non-finite quantity.

Beginning with the examination of the latter view, let us suppose, firstly, an infinite discrete quantity. Such a one, of course, is not finite. Furthermore, it is not commensurate with any finite quantity. But what an awkward idea is this! We seem to have a principle generating a series (I mean of numbers) covering any possible discrete multiplicity; and yet we wish to provide the means of measuring an imagined discrete multiplicity not covered by that series. But the series is meant to be and is all-embracing, for it is without end; it is its infinity indeed which guarantees its comprehensiveness. And even more, we do not seem to mean by calling the series 'infinite' anything more than its adequacy to measure any possible discrete quantity, and, surely, all actually existing discrete quantities.

But some dialectics may also be of help here, though (as always, according to the nature of dialectics) only negatively and propaedeutically, by removing some stumbling blocks, by loosening connections commonly thought unseverable etc.

According to the view under discussion, although the conception of a greatest finite number is illegitimate, this is not so with the conception of a greatest number simpliciter. For clearly there is a greatest number: the infinite; and it is greater than any finite number, though there is no limit in the ever-increasing series of finite numbers. Obviously then, atension, perhaps unresolvable, is generated here: the claim that something infinite is greater than anything finite points to an ultimate homogeneity of infinite and finite, whereas the further requirement of something's being

greater than anything belonging to a series without end (of any member of which there is a greater), indicates (apart from its incongruity) a fundamental heterogeneity between it and items of the series.

Further: if x is greater than y , then x is greater than y by a certain amount. Now exactly how much is the infinite greater than the various finite numbers? Not by any finite amount; for by adding any finite quantity to any given finite quantity we get as result always a finite quantity. Then by the infinite? (For there is one definite, infinite quantity). But then one and the same definite quantity (the infinite one) differs from any finite quantity by one and the same amount. Which is clearly absurd.

One more example, still playing this game of dialectics: Any quantity has parts. And any discrete quantity consists in a certain amount of units, which can be variously arranged so as to yield different parts of the said quantity. (I mean that, for example, $4 = 1+1+1+1 = 1+3 = 2+2$). That is, any given discrete quantity has certain numerical relations to the unit and to other numbers as well, in respect, here, of addition. Now an infinite quantity must consist in an infinite number of units. But according to the supposition, an infinite number of units is one certain definite (though non-finite number of units. Hence, equally well, we can conceive of the same (an infinite) number of twos instead of units. But what now is the resultant quantity? Not the same as before; for, surely, a definite number of ones and the same definite amount of twos cannot yield one and the same quantity. It must be then a different one; but obviously the new one must also be infinite; yet, we have supposed that infinite is just a certain (one and only one, definite) quantity!

32/ We can multiply examples, if we wish to. But such 'proofs' in philosophy are, I conceive, just part of a dialectical technique whose function is mainly 'educational' in that it must be taken as aiming to help people see the principle through its application, the general in the particular. (Of course a connection with Aristotelian *Gopūjī* is meant here). Anyway, the above given examples suffice for our purposes here, as it will be seen in the sequel.

Let us then turn to the case of acknowledging many infinites with an eye to see whether we can thus avoid absurdities like the above ones^I. Suppose then that we postulate a basic infinite number ∞ , and then, in trying to overcome the second of the above mentioned difficulties, we pose ∞_n (with n a finite number) as the infinity comprising an ∞ number of n s symbolising this by $\infty_1 = \infty n$. Of course we would now have: $\infty_2 = \infty \infty_1$, $\infty_{\infty} = \infty \infty_1 = \infty \infty n$,

$$\infty_{\infty} = \infty \infty \infty, \quad \infty_{\infty \infty}, \quad \text{etc.}$$

∞ times ∞

But we have also to solve the first noticed absurdity. So, let us begin with any one of the introduced infinites, for example, ∞_1 . It is: $\infty_1 = n + n + \dots + n$, where ' ∞ times' means ~~ex~~
 ∞ times hypothesi a certain, definite number of times. Now it is clear from the above formula that ∞_1 is greater than n by $(\infty - 1)n$. But then we are forced to multiply still more our infinites, even beyond the limits of the generous multiplication found necessary above.

And still worse is to follow. For we have got to have $\infty - 1$; let us call it ' ∞' . Then why not $\infty = \infty - 2$, $\infty = \infty - 3$ and so on. But if $\infty = \infty - 1$, then $\infty - \infty = 1$. And similarly, from the other equalities, $\infty - \infty = 2$, $\infty - \infty = 3$, etc. Further $\infty - \infty = 0$, for if from some one definite quantity we subtract itself, nothing remains. Hence we found ourselves having constructed a series:

$$\text{I/ } 0, 1, 2, 3, \dots, \infty, \infty, \infty, \infty.$$

Surely we can add further to it. For $\infty = \infty + 1$,
 $\infty = \infty + 2$, etc. And: $\infty = \infty + (\infty - 3), \dots$

$$\infty = \infty + \infty = \underbrace{(1+1+\dots+1)}_{\infty \text{ times}} + \underbrace{(1+1+\dots+1)}_{\infty \text{ times}} = 2 \cdot \infty = \infty \cdot 2 = \infty^2$$

Hence series (I) becomes:

$$\text{I'/ } 0, 1, 2, \dots, \infty, \infty, \infty, \infty, \infty, \infty, \infty, \infty, \infty, \infty = \infty^2.$$

We can, in an obvious way, continue the series analogously.

I. It might appear very curious indeed that I should have even

And now, surely, it is not the finitist who should be surprised from the manifest result. For granting a positive nature to a fundamental infinity and taking seriously into account the requirement of its definiteness, we have seen it behaving like any finite number 'no matter which', behaving like a finite number left unspecified, but not for this reason considered as indefinite and indeterminate. This result is obvious from series (I') and its natural extension. And that is why we have selected the two absurdities of 3 31, from an attempt to meet which we were led to the construction of series (I').

33/ We conclude then, that an attempt to take seriously into account the requirement of definiteness (though not finiteness) of infinity, leads directly to the assimilation of the latter with finiteness and its absorption into it. Consequently we remain wherefrom we have started: infinity (referring to discrete quantity) is lack of determination in respect of number; or, in other words, the concept of infinity is a privative one, signifying either the privation of numerical determination or the 'matter' qua numerically undetermined---corresponding to whether we are thinking of infinity itself or of something¹ infinite.

34/ The above procedure may be accused as committing the fallacy of petitio principii. Thus, it might be said that I have employed some principles which are valid only in the domain of finite numbers and numerical quantities². Such principles are:

consider at all such extravagant supposition that there is just one definite, non-finite discrete quantity or number. Nevertheless, the idea in itself (apart from its consequent absurdities derived from it) is at its face value less repugnant, if possible, to reason than the idea of there being many or infinite infinities or hierarchies of infinites. Indeed, in ancient philosophy normally an infinity of infinites is felt as aggravating the situation of which it is proved, even beyond the point reached by a single 'simple' infinity. We feel almost conversely, mainly, I suppose, under the influence of modern mathematics---or rather of the quasi-philosophical interpretations of modern mathematics.

1. If not an entity, anyway some kind of hypostasis however remote from anything concrete or abstract. I hint, of course, here to a combination of Aristotelian and neoplatonic metaphysics---of which more in subsequent essays, when the notion of matter will have been introduced.
2. Compare, for example, with Russell's attitude in this connection.

- i/ discrete quantities differ by certain definite amounts
- ii/ one and only one discrete quantity differs from certain discrete quantity by a certain definite amount
- iii/ a discrete quantity consists in a certain definite amount of units
- iv/ the subtraction from a discrete quantity of itself leaves nothing.

Now my plead regarding this objection is that, as it seems to me, the above principles depend for their validity not on the finiteness of the discrete quantities subsumed under them, but on their definiteness? And if so, it was correct that they should be put into use in criticising a view according to which, the infinite is just a kind of (definite) discrete quantity (only a different kind from the finite one).

(The ultimate principle was of course, a form of the Law of Determinateness: an indefinite quantity is no quantity, but only, at most, potentially a (definite) quantity¹.).

35/ But it may be objected:

"Even if there is an intuitive plausibility in holding that the principles (i)-(iv) of 35 are valid in virtue of the definiteness of quantity and do not depend on finiteness (except that by use of them we come to the identification of finiteness with definiteness), still the principles must really depend in the last analysis upon finiteness in the sense that they cannot be considered as holding good for all discrete quantity; for after all there exists a body of doctrine in mathematics relating to hierarchies of infinities. So that it just is possible² to speak intelligibly about infinities of various orders in mathematics, in the science, that is, of quantity, and to speak so without abolishing the essential characteristic of mathematics as a science of quantity. Therefore, presumably, there is nothing wrong with the conception of

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- 1. Strictly speaking, we ought to formulate it in terms of quantified things rather than quantities, but sometimes freer ways of speaking are not substantially harmful in any important way.
 - 2. And it is possible, since it is actually done.

infinite (or transfinite) quantities, provided of course that we keep them distinct from finite quantities and are cautious enough not to infringe on their realm with principles drawn from and valid in the other's territory?

Now this objection is illuminating, in that it exemplifies a kind of argumentation which I cannot but consider as philosophically suspect, and of not great philosophical importance, if of any relevance----at least if taken at its face value. For, in effect it objects against a philosophical body of intuitions cum reasonings, on the ground of a certain description of what is going on in a given positive science---of what operations and procedures are (adopted as) valid in it. In this way a seemingly unresolvable tension is exhibited between what we are inclined to hold on philosophical grounds and what we in fact (are said) to hold, at least implicitly, given what we do in a certain positive science. For example, we have discerned much philosophical plausibility in the view that infinity is a privation, and not a definite quantity. But we operate with a positive notion of infinity in (certain branches of) mathematics, and in this way, so we are told, we hold that infinity is something positive, meaning transfinite (yet determined) quantity. The tension is ready at hand; it has been constructed.

36/ Before resolving the tension in the obvious way in which it is resolvable, let me try to locate the phenomenon in a broader perspective.

According to a widespread 'diagnosis' of the philosophical 'disease', the intellectual disorder which supposedly constitutes traditional philosophy arises from not paying proper heed (attention) to the actual functioning of concepts within the discourse, into which they are naturally embedded. The idle contemplation of our conceptual implements out of and in abstraction from their natural employment in the proper field and according to the normal way, produces the "I do not know the way out" characteristic feature of the philosophical puzzlement. For the philosophical 'must's or 'cannot be's, figments of the vacuous, as it were, working of the mind, are forcefully contrasted and opposed by the 'is not's and 'is's, respectively, of its down to earth observation of the actual

functioning of the machine under proper 'loa d'---so to speak.

We can generalise then to what may be called the Idlesse theory of philosophy. According to it, we have on the one hand the working days of the everyday transactions and the properly scientific investigations, and on the other hand, sharply distinguished from them, the vacation days of the philosophical (in the traditional sense) 'speculations'.

Of course there are varieties of the above 'theory'. There are some who would consider the intrusion of science in the above scheme as irrelevant. And others who, conversely, would find ordinary 'argumentation' as an unwelcome, to put it mildly, partner to scientific investigations. But still, admitting the differences of the variations, we can also see that at least one attitude is common among them; in one or another way they consider philosophy as lacking both a proper to itself subject matter and a methodology (in the broadest sense) peculiar to it, a methodology of dealing with its subject matter, whatever it would be (or would be taken to be)? Speaking generally, they view philosophy as a tragic-comic attempt of working idly with a mechanism adapted to work in some definite field and therefore suited for such a work; or of working with such a mechanism in imaginary fields, or in real admittedly and legitimate fields, though different from those to which the mechanism in question is adapted.

And indeed, as a description of some kind of philosophy the above sketched account is not at all bad (specially in its last clauses)¹. But, unfortunately, it so happens that advocates of views like the above, not unusually provide us with some of the most interesting and striking instances of such a kind of philosophy; they just fall in the trap, in which, according to them, traditional philosophy has fallen. For example they may try to persuade

I. I refer for example to the various scientisms; attempts to model philosophy on physics, biology, psychology, economics, linguistics etc. But we must be very careful here, as anywhere, in allocating labels. For there may be that the modelling is external and superficial (the produce, for example, of a preoccupation with the formal method of a certain science); or it may happen that the modelling is substantial, say the application of principles peculiar to one field of study, to another discipline. So it may be the case that, for example, Spinoza or Proclus are much more 'purists' in philosophy (notwithstanding their apparent preoccupation with the formal method of geometry), than, say, Hume or Austin, who do not betray such an attachment to some one or other Form of method.

us that philosophical worry is to be solved solely by paying minute attention to the actual usage of words in ordinary language; or by finding its psychical source; or by reverting to the sociological or economic situation of which it is an expression; or by many other curiosities of this sort.

Any way I shall not argue against the Idlesse Theory of Philosophy here. It suffices to show common ground among some seemingly vastly divergent conceptions of philosophy, and to oppose those to a philosophical purism, according to which both a subject matter and a proper methodology exist which are the absolute prerogative of philosophy, and which are to be extracted from an unprejudiced and unbiased close study of the philosophical Tradition (unprejudiced: not working hermetically shut in the scheme of contemporaneously fashionable methods and ideas; unbiased: not trying to read in tradition the above mentioned methods and ideas with the circumscribed subconscious thought that in this way we justify tradition^I).

37/ Returning to 35. The concept of infinity in mathematics presents in itself indeed various problems. But how the idea may have occurred that this concept must be the same with the concept of infinity in philosophy? For only under such a presupposition is the tension supposedly discerned there a real tension.

We on the contrary wish to claim that regarding both subject matter and methodology, sciences differ from philosophy. And hence their problems are different, in spite of a possible but accidental absolutely speaking (though quite understandable historically) similarity or even identity of formulation.

Compare for example, the problem of matter in philosophy and physics; or the problem of life in philosophy and biology; and the problems of number in mathematics and philosophy; or the problem of predicate in ordinary grammar, in scientific linguistics and in logic---or rather we must further subdivide here mathematical or scientific logic from the philosophical one; for from the end of the last century, logic has suffered the same kind of bifurcation as all other initially unitary disciplines.

I. We must understand tradition in its own terms, not by imposing our own. To put the whole affair briefly, but because of this regrettably provocatively, it is not Tradition which has to be justified; it is we who have to be justified on the face of Tradition.

But one must beware of a trap here. For the historical fact of the development of a science out of a philosophical discipline does not mean that the only way in which the 'mother' discipline can continue to exist after the separation is by being transformed (or, allegedly, by being made more precise by being transformed) to a philosophy of the corresponding positive or mathematical science. So natural philosophy by 'begetting' so to speak, mainly, physics is not ipso facto reduced to philosophy of physics, in the sense of being a general and theoretical investigation concerning the conceptual & framework and methodology of physics. Indeed, we would like to say that there is as much place for Natural Philosophy proper (as distinct from Philosophy of physics) after Renaissance, as there ever was, ---and rival theories compete in that place (as different as Aristotelian or Hegelian or Russellian in character), even as, perhaps, Quantum Mechanics and Wave Mechanics compete on the ground of physical science.

38/ It is perhaps fit to remark that in most fields of mathematics the use of 'infinite' exhibits something of the negative attitude above sustained on philosophical grounds. So it might be said of a series not converging at any finite number and of a certain kind, either that it converges at the infinite, or alternatively that it diverges (to the infinite). However, the notable exception is that of the theory of sets, whose hierarchy of infinites was described already in the nineteenth century and within mathematics as the 'Mythology of Mathematics'.

39/ But, one may say, even conceding that one cannot draw directly from certain formulations of what is going on in, for example, set-theory, conclusions of philosophical importance (or even interest or relevance) concerning philosophical problems, still one has not to make mathematical in the strict sense (i.e. relating to a certain formulation of a theory about the mathematical body, so to speak, or subject matter--say, numbers) assumptions in order to see that just a serious acceptance of classes suffices to necessitate the acknowledgement of some form of even actually existing infinite. For, the objector may go on, begin with any finite collection of

objects. Then, surely, the class constituted by such collection is something distinct, over and above the objects comprised in it. So that if we have had n objects, we must acknowledge $n+1$ entities. And these form certainly a new class distinct, over and above those $N+1$, entities. Which raises the number of existing entities to $n+2$. Continuing in this way, we naturally find ourselves having to admit that really a^{∞} infinite number of entities exist in some way or other even if we began from n initially acknowledged objects. (Indeed . . . we might begin with just one object; or with none, for that matter---according at least to some conceptions of the problem). The prospect of piling up infinity upon infinity is again ready at hand.

40/ But, of course, I should reply, the argument can be turned upside-down. For precisely because its conclusion (the actually existing infinite) is unacceptable, it must be invalid. And indeed the problem is exactly to 'diagnose' its invalidity.

(But still, out of curiosity, one may naively ask: "Well, and when we have reached infinity, or first order infinity at any rate, what is the next step? I mean what is the multiplicity of the class of all infinite entities 'discovered' till now plus the class of these infinite entities? Is it the answer ($\infty + 1$)?").

i/ One may find fault with the notion of a collection of basically (categorially) different members, and speak correspondingly of illegitimate collections used in the above argument. I do not think that this reply cuts deep, but I shall return to it after developing a second possible diagnosis and reply.

ii/ There is no distinct entity (the class) corresponding to a given collection of objects. A class is just the objects \neq 'forming' it, is just its members^I. This claim, perhaps provocative if boldly and summarily announced, is nonetheless, I think, evident, provided we keep clear in mind the following distinctions:

- I. One may be prone to object in a rather rash and naive way: "But if it has members, it must be something". But from the fact that we ordinarily describe the situation by using the locution 'a class has members' nothing philosophically relevant follows ipso facto; hence it is not significant that I avail myself of this locution in a preliminary statement of the idea I wish to convey. On the other hand, of course I must say after the completion of the analysis, what the proper expression would be on my view---if the proposed theory aims at completeness,

a/ a class of objects qua class is not a complex of objects, neither a whole of parts. (Naturally a complex of objects is something distinct, over and above its elements, for it is the elements structured, arranged, in a certain way, in the way so as to constitute it).

b/ a class of objects is not, obviously, a common feature or property of a given collection of objects. And further, neither do classes of objects necessarily stand in an one-to-one correspondence to such a property (or properties) in the sense that for any given class there is (necessarily) a certain feature(s) which is(are) such that all members of the class and only members of the class are characterised by this(these) feature(s). Nor do classes of objects necessarily correspond to properties in the sense that all members of the class are characterised by those properties (even if non-members are also so characterised). Nor even do classes of objects universally (if not necessarily) correspond (in either a one-to-one or in any, indeed, way) to properties in the sense that for any given class there is a property (or propertise) such that all members of the class (let alone; and only these) are characterised by this (these) property(properties).

Two objections may be urged against the above claims in (b) (which claims are, I should say, backed by and based on intuitions ---as it seems to me).

bI/ "Granted that there is no universally ---
not to say necessarily, holding one-to-one correspondence between classes of objects and properties in the a bove explained sense, yet there is indeed necessary connection between any given class and some certain(maybe very general)common feature of its objects. For a class is a class of 'somethings', a class, say, of Us. And then U-ness is the required feature----even if it is just the 'character' of being an entity or even objecthood itself".

There are more than one issues entangled in this objection, and it is, I think, worthwhile trying to disentangle them.

(1)and(2): We have(so far) discriminated five most general categories(highest genera)under one or other of which everything examined till now allegedly falls. These are (2 10)

the categories of (atomic) simple fact, concrete thing, abstract determinant, instance and tie (subsuming under it the three types of tie distinguished). This classification is not propounded as anything near a complete inventory of the most general kinds of everything¹. But provisionally accepted will do for our present purposes. Now the important question to be asked is, Have all these categories or some of them anything in common²?

For we may be inclined to say that concretes and determinants are things (but not items in the other categories are things --in the philosophical use of the word); that concretes, determinants and instances (and possibly ties but not facts?) are entities; and that any item in any category is an object (conceived very broadly in a special philosophical sense having some correspondence with the 'colour' and overtones of 'objective'). And so far so good; indeed it is very convenient to avail ourselves of those linguistic (appellative) distinctions, for by so doing we 'naturalise'—so to speak—our formulations by not mentioning, for example, continuously 'everything', and by removing some of the 'weight' of careful

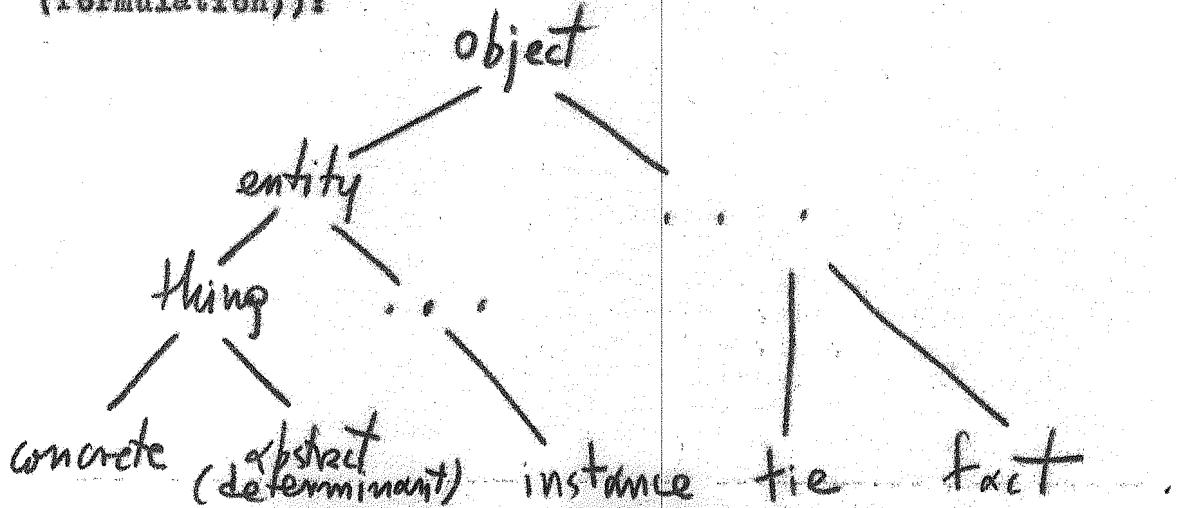
1. The formulation is deliberatively elusive. For I do not want to say "of everything existing" in order on the one hand to avoid begging the question regarding the various modes of existence and subsistence, and on the other hand to account for everything possible, too. Perhaps the expression "of everything possibly or actually existing or subsisting in any way whatsoever" will do as suggesting the utter generality and hence, in a certain sense, the utter vacuousness of the desired formulation. For other reasons of the elusiveness see the continuation in the text.

2. This is the philosophically relevant issue, which underlies formulations which I deem to be misleading precisely in so far as they keep this fundamental issue in the background, out of the foreground; I have in mind formulations like: "Is it proper that all or some of these categories should be called 'entities' or 'objects' in general etc.?", or even worse: "Do we ordinarily apply the term 'entity' or 'object' to items under all or some of these categories?".

circumlocutions¹. But we must beware of confusing linguistic conveniences or even proprieties(2) with attribution to items of common characteristics or natures(and which) (I), signified by our convenient or not, fitting or not expressions.

(3) A third issue is the one concerning whether it makes sense to speak of a certain number(or collection and, hence, class) of items only under the condition that they share a common essential nature², (determined further in some cases by some other accidental characteristics).

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1. Therefore we may provisionally work under the following scheme(considering it as terminologically convenient, and also as doing some justice to what *prima facie* we are inclined to hold in respect to the classification at stake---to hold, I mean, as a result of what we *prima facie* recognise as our intuitions regarding the studied field, and what we *prima facie* consider as their adequate and proper expression (formulation)):



2. I say 'essential nature': for, so it seems, it does not make good sense to speak of a collection of red (red some things) if only the condition that we have to do with items which are red is fulfilled; it is further required that these items should be of a certain substantial kind, should belong to a certain species or genus, in respect of which, mainly, they are counted(of course taking also account of their accidental characteristic)For concrete things of various kinds can be red, and also parts of them, and unities out of many concrete things touching one another and so one by contact, and surfaces of them, and parts of their surfaces; and even perhaps instances of redness are red. And even if we mean 'a collection of red concrete things' and also deny that instances of redness are red and further 'accommodate' surfaces so that, finally, only concrete things can be(at least primarily)red,---it is still true I think to say that redness itself does not provide the required unit of counting, the required principle of individuation, the principle of 'defining' and counting individuals; this is the prerogative of the substantial component. And indeed we may be touching here an important difference between what I call 'proper predication' on the one hand and U-predication(predication in the category

Let us now, by bringing all three issues to bear upon the objection (bI), see what is correct in it.

Of course we speak and, ordinarily, conceive of a class as a class of 'somethings', as a collection of items, of objects (in our preagreed use of 'object', relating to any item of any category in the 'universe of discourse' so far examined). But the problem is precisely this: what is really involved in that conception? What more is contained in it than just the obvious, though trivial, requirement that in order to refer noncommittally to the members of a heterogeneous (from the lesser steps of heterogeneity up to categorial heterogeneity) class, and do this not through the description 'members of the class in question', we must use and employ terms consistent with the envisaged heterogeneity and not properly ascribed to just some of the heterogeneous members of the class. For this is an element in our conception of a class as a class of 'somethings': I mean the use of terms consistent with or comprising, so to speak, all diversity exhibited in the members of the class. But we feel, something more, and more important, is included in our conception. And the important problem is, What more?

And now the affirmative answer to issue (3) above presents naturally enough itself as the solution to this last problem. For, on ϵ may say, the additional element (or elements) required is:

(I) that all members (though not only these necessarily) share a common nature,

and (II) that this common nature is "individutive" or "unitative" in that it provides a principle of units or unitary individuals or simply of individuals (using the term so that 'unitary individual' is a pleonasm—compare with $\delta\tau\mu\sigma\nu$) for objects exemplifying it.

This sounds plausible enough. But is it?

Suppose we have two concrete things a and b, and two (abstract) determinants P-ness and Q-ness.

of substance) on the other. (Of course this account leaves predications of the sort (A) 'this is (a piece of) gold' standing in need of a separate explanation. We may, however, already see the direction to be followed for an answer to this: (A) may be construed either as 'this is golden' or as 'this is a gold-piece(a golden-thing)').

It is to be noted that I have switched from 'essential nature' to 'substantial' in the course of the above exposition. But what this transition indicates is nothing more than a shifting of interest from the general point about any item in any category whatsoever, to the particular case of object under the category of concrete thing (or proper substance).

Suppose further that there is nothing common among all or even some of the categories sketched above. I am not saying that this is so, but I am asking whether in case that it is so, we could form no conception (no conception at all) of the class consisting of a, b, P-ness and Q-ness, as a result of its being so¹. Viewed in this light, an affirmative answer seems, I trust, awkward. And I think that we can explain its queerness by positively propounding another (but connected) account of the additional element required to supplement our terminological condition which must be satisfied in order for any objects to be possibly conceived as belonging to a class.

And this account is as follows: the additional element is:

(III) that all members exemplify unitative(individutive) natures---though not necessarily one common unitative nature. That this is necessary condition for the conception of any class, is obvious. The point is that it is also sufficient; that we need not look for anything more. The concept of class is necessarily connected (it involves) the concept of its members²; a class has members; and members of a class are certain items qua objects considered as belonging to the class. But certainly these items must have a unity of

1. If the matter was in fact so simple, that is, if the participation in something common was a prerequisite for objects to be possibly conceived as belonging to a class, then we would have in our disposal a relatively easy way of ascertaining for any given objects whether they have something in common: namely by seeing whether we can conceive them as belonging to a class!

2. Indeed to have a conception of a class is necessarily connected (involves) with having a conception of its members. Obviously not an actual, explicit apprehension of all its members if the class is defined through a common property (or common properties), for example. But, anyway, this would be a case in which the members of the class in question do have a common characteristic, and, hence, it is irrelevant to our present issue of whether this is so necessarily and for all conceivable classes. In such a case, our requirement (III) would demand that a principle of unity and individuation is at any rate directly or indirectly provided through the common characteristic ('indirectly': I mean through a suppressed element like "concrete thing").

Besides, when I say that the conception of a class is necessarily connected with the conception of its members, I mean rather that even in cases in which an actual, separate cognition of its members does not take place, I nonetheless have the conception of a collection of objects satisfying the defining characteristic(s) of the class---i.e. of the collection of members of the class.

some kind or other, must be "ones"¹; this is required by the discreteness, so to speak, by the distinctness of the members of classes---and of items in so far as they are qualified to serve as members of classes².

1. It might be held that it is one thing to be one or a unit, another to be unified or have a unity. But how are these notions supposed to be differentiated? In as far as something has a certain unity, in so far it is somehow one and in so far it is possible for it to be taken as unit; and conversely, in as far as something is one 'something' and can serve as unit, in so far it must have some form of unity. (To anticipate an objection from the neoplatonic metaphysician, I must make clear that in 'something is one', 'one' is used adjectively or predicatively, not denoting the One or oneness as in a statement of identity.).

2. One may object: "What contrast is the last clause intended to mark? Which items are not "ones" (in some way or another "ones")? Indeed, in as far as we can conceive of something, in so far we conceive it as a one-something; and conversely." This is an important remark, but not, I think, a fair objection.

The importance of the remark lies in that it brings to the foreground the problem (to put it summarily) of the relationship between Being (of whatever kind and in whatever sense) and Oneness (or rather, in my own terms, the relationship between Objecthood and Oneness); in all its various aspects. And a direct confrontation with this problem is essential for any adequate metaphysical system sooner or later. We can observe exemplarily the centrality of the problem in the Neoplatonic Metaphysics, also we can better understand it as well as its importance through a co-examination of the divergent treatments given to it by, say, Aristotle and Neoplatonists. But however important in itself, the remark is not very much relevant to my point in the text, which was the negative one of disqualifying the view that the unity of class-members must be established on the basis of a principium individuationis correlated to a common nature shared by all co-members of a class. Consequently, I was only trying to show that for items, their being separately one (their oneness or unity, the 'fact' that each one of them is one) is a sufficient general condition of their availability for class-membership, and that therefore the oneness in question need not be correlated with a shared common nature. And further I was trying to establish this, independently of a positive solution to the important problem mentioned above.

On the other hand, let me mention, no particular solution of this problem invalidates my general point in the text---and this is the justification of the possibility of independent treatment. For, to take the obvious candidate for such invalidation, I am indeed inclined to hold that anything, any object at all, is one just qua (in so far as it is an) object, that any item whatsoever is one (and not only those items qualifying for class-membership are so), that it is one on account of its being something, an object, that its oneness is intrinsically bound to its objecthood. But this is not really against my claim regarding the conditions imposed on items for class-membership. For I can retort firstly that, even so (that is, even if objecthood and oneness are intrinsically interwoven in the way suggested),

However members of a class may be ones in various ways (if not senses), the one being one X, the other one Y etc. There need not be for any conceivable class some corresponding nature X, such that every member of the given class must be one X. (In the modern idiom; this requirement is not included in the concept of class).

still classes could be formed and conceived even if this was not so, in the sense that class-membership itself does not require the fulfillment of this condition on the part of object-members. In other words, that condition is something which it holds, if it holds, of object-members qua objects and not qua members. And I can retort secondly that from the above view (to which, as I have said, I am nonetheless inclined to subscribe) it does not follow that 'one object' is strictly univocal if applied to items of various categories. It may mean one X if applied within the category X and one Y if applied to the category Y----and only 'one X' and 'one Y' would be, on this account, strictly unequivocal. (Of course there may be close connection between 'one X' and 'one Y'---for some categories or even for all categories: compare with the case of focal meaning. *πόσις ή πλέον δύο* and *τοῦ ένας λέγεται οὐκ*). In other words, it may be the case that everything is one, and yet be one not strictly speaking qua object, but qua X-object ('X' designating a category as above).

Now admittedly this last point does not flatly contradict the objection from which we have started in this note. But, if I am not mistaken, it "steals away" so to speak, its point, and much belittles its force.

Anyway, this same point is on the other hand in itself very interesting and capable of introducing us into further refinements. Just in order to hint at the complication of the whole problem rather than to offer positive solutions, I may make the following two further points.

i/ We remarked that from the intrinsic connection of objecthood and oneness it does not follow that 'one object' is univocal (the connection is also expressed by saying that anything and everything is one object). But now, on the other hand, we must remark that from the alleged equivocality of 'one object' it does not follow that 'one' is equivocal if applied to items belonging to different categories. It may be said, for instance, that the equivocality of 'one object' is duly borne by the equivocality of 'object' and that there is no need for infecting 'one' with it, too. This is indeed a usual enough form of argument. Now we must distinguish between the general claim and the particular application of it. As general claim I mean the view that if an adjective (adjectival word) 'A' applies to objects both of N_1 kind and N_2 kind, then 'A' need not be ambiguous; it may of course be, but it need not be, just and only because it applies to objects of different kinds. Again, this general claim is susceptible of two interpretations: interpreted weakly it is claimed to hold good under the condition that N_1 and N_2 belong to one and the same category; (other restrictions may be imposed, such that, for example, the difference in kind should be of a certain kind, not very fundamental so to speak etc.); interpreted strongly, it is supposed that no such restriction ought to be imposed and that it holds unqualifiedly.

So we can see now what is valid in (b1): of course it is true that usually or ordinarily members of the classes which we find occasion to conceive are pretty homogeneous---indeed they share a quite specific and rather determinate property. Further, of course it is true that even in extraordinary cases, when we conceive of extraordinary classes, we can refer to their members by using words like 'thing', 'entity' or (if even these prove to be inappropriate according to our stipulations---like the one in n. I on p. 37) 'object'. But these words may be contrivances of neutrality as among common natures, not indicators of further higher common natures, posited above the categories. And, finally, this being as it may, classes and members could be conceived even in case that those words were not signifying common natures shared by items to which they could correctly be applied.

b2/ "There is indeed for every class C a property such that every member of C, and only members of C, have it: namely the property of being a member of C".

This, of course, seems to be a rather shallow objection, especially if compared with (b1). But it is negatively, pathologically as it were, interesting because it can be faced from a number of quarters. We shall ^{look at} it herefrom a general viewpoint, independently of our considerations about the notion of class.

Of course, it does not follow from either form of the general claim that 'one' is univocal. And At most, it leaves the matter open. And this quite apart from the consideration that we must apparently recognise some special status to 'one' among adjectival words---to put the point in the linguistic idiom.

ii/ Now again, from the circumstance (if it really obtains) that 'one' is univocal it does not follow that there is someone, very general indeed, (positive) nature denoted by 'oneness'. Not that this 'inference' is in principle nonsensical; it simply is not always valid. For 'and', for example, may be said plausibly to be univocal, without, obviously, implying anything as to a certain nature. And 'not-red' may be said with good reason to apply synonymously to all coloured things which are not red, but surely it does not signify any positive nature, being a privation (in the above sense, when applying to coloured things; it is simply a negation, when applied to objects not susceptible of colours). And besides, 'one' is clearly an exceptional word, too.

But now, as a final touch on the rough drawing presented in this note, let us suppose that someone is prepared to admit a positive nature corresponding to 'oneness'. This must be an all-embracing, highest nature, since we have granted that absolutely everything is one (something). That person then proceeds: "All members of any class share after all a common nature: oneness. And you have admitted that member-objects must be ones qua members. Hence you have been refuted".

My answer is simple: this would not do, even if we accept all the premises of the objection. And we have caught a glimpse, even from the above preliminary sketch of the field, of how ma-

Whether a determinant, an abstract thing, characterises or not a concrete particular (in case that it is a relation, relates two or more concrete particulars or one particular to itself---but I shall not mention this separately, including it into characterisation or determination) it makes a difference to the concrete thing itself. The concrete thing is different in case that the determinant characterises it from what it is if it does not characterise it; and different in respect of that determinant, and qua characterised or not characterised by it. And indeed, if the determinant is essentially predicated of it the mentioned difference is still more widened: the concrete thing would not even exist if the determinant did not characterise it (given that the determinant actually and essentially characterises it), or a concrete thing of a different kind, and not simply a different thing, would exist in its place.^{1,2}

issues such an acceptance must presuppose as being solved). For even if there is a positive nature denoted by 'oneness', it is not unitative-individuative in the above explained sense. This sounds admittedly paradoxical, but, yet, I do not know how to avoid it: oneness itself does not provide a principle of counting individuals 'characterised' by it; it is, so to speak, the principle or general form of all individuative natures in respect of their individuative force, but it itself is not such a nature. And my claim in the text was that members of a class need not share one common individuative nature, which constitutes them as discrete, distinct members of the class.

4.

It may be said that this is not generally, let alone necessarily, the case. For what of A's property of being famous? Or of being loved by B? Or of being seen by B? It does not seem to be the case that necessarily the having or not having of such properties implies or results in some intrinsic difference of the particular characterised or not characterised by them? To which objection the reply must be that loving is a binary relation; hence in each of its applications it must characterise two concrete things capable of bearing it. (I avoid here complications regarding the possibility of one thing's loving itself, or regarding construals of loving as a 'three-place' relation 'A loves B in respect of C' etc.) And so the required difference is in respect of both these two things. To put it tautologically, whether a relation characterises or not makes a difference to the way things are related, and it is in this sense in which it makes a difference or affects the things involved in it. Or if you want, it makes a difference and affects the things involved considered collectively as forming a certain totality.

A similar analysis is ready at hand for being famous, being seen and similar relational properties. It is further to be noted that the holding or not holding of a relation may result to some real difference to some of the things related. The point is only, that it necessarily affects the totality of the things involved. Anyway an adequate treatment of these problems, require a developed theory of Determinants.

Now what of this alleged property of being a member of C? What difference does it make to a thing (how is it different) the circumstance that it has or lacks this property? None at all, at least none necessarily. And if, in any case, it does correspond to some difference in the thing itself, it is because the class in question is defined and constituted by some genuine property, and therefore that a concrete thing belongs to the class or not, is just dependent on whether it has or lacks the said genuine property.

So that the 'property' of belonging to a certain class is in itself (I mean quite apart from the possibility of indicating the having of a genuine property) a pseudoproperty, not a proper property. Hence the objection falls down.

But I have said that the objection is interesting, if only pathologically so. And it is interesting, *inter alia*, because by calling for a clarification of the notion of being a member of a class it points, ultimately to the direction of a positive analysis of the notion of class itself. We may put the problem briefly as follows: if 'being a member of a class' does not signify a genuine property, then what sense does it have? For, of course, we do not wish to imply that it lacks sense! (The usual, easy short-cut).

We have denied (and try to justify our denial) objective existence to classes as objects distinct, over and above their members. There are no such objects, i.e. objects corresponding to collections of objects in such a way that these latter are members of the former. But surely, on the other hand, we can and do conceive of classes of objects. Certainly we do; because we ourselves construct classes. The conception of a class is the construction of that class.

There are objects. And we wish to some of them, sometimes, together. We form then a class of those among them which we wish for various reasons to consider together. We collect mentally them; and in so doing, in just this our co-consideration of them, in this our conception of them as being together (for our purposes), we construct a class with those objects as members. To put it briefly:

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The expression 'in its place' can be further explained by reference either to the notion of matter, or to the notion of a possible concrete thing. As to the latter, compare with the analysis of this notion given in Essay III in combination with the there introduced notion of incompatibility in existence among possible concrete things.

classes have an epistemological, not an ontological point¹. Or if one wishes: classes have subjective, not objective existence.

But let us proceed further. Granted that classes are mental products, still in being mental products they are somehow. And in an adequate metaphysical inventory they cannot be just put aside; the question arises: to what category do they belong? Where do they fit?

Now, of course, this problem is a particular case of the general problem of the correct metaphysical assay of the mental (and of its various kinds); or of the problem of mind. For one way of answering the problem of classes along the previously drawn lines, would be to say that classes are modifications of the thinker or of the thinker's mind. And in a sense they are. Yet this does not seem to be the whole of the affair, making, as it does, them too subjective². Like the case of knowledge, we cannot acquiesce completely in holding that they are just mental states or processes. And this our troubled conscience is perhaps making his way when we tend to describe them as mental products rather than as mental processes. Products seem to have a certain independence from their originator, even from their origination and production.

I suggest (but shall refrain from developing the idea here) to consider construing the notion of class as the concept of a certain relation \in holding between persons (or minds) and objects of whatever kind or category. For example: 'ACC₁, o₁, o₂, o₃' means that A 'co-classes' o₁, o₂ and o₃.

Now, if this idea really works, we can understand and explain why the belonging or not of an object to a class does not make a genuine difference to it (and if it corresponds to such a real difference, it is because of the way the relevant class is defined).

1. I may draw attention to the similarity of the treatment of classes here and the treatment of negation, disjunction and conjunction in Essay IV.
2. For example there would be different classes of the same collection of objects, corresponding to the different minds conceiving of those objects, or even of the time at which the conceptions take place. But we may, of course, say that the classes are objectively (as objective entities identical with their members) numerically the same in so far as they consist of the same objects, but subjectively different (as mental groupings, or products of such groupings) in the above referred cases.

----and hence is not a genuine property. For 'being a member of a class' means 'being co-classed together with other objects by some person(s) and hence clearly the difference is borne by the totality of the objects involved in the given relation, and not necessarily, by just one object-member; that is in the above example, the difference made does not fall exclusively and separately on, say, o_1 , but on the way it is related to A, o_2 and o_3 .

It is quite possible to elaborate further the above idea, but I think enough has been said in order to be justified in holding that a positive account can be found (even if the correct positive account is different from the one above indicated), which would enable us to do justice to all our intuitions concerning the studied nexus of problems without invoking the objective existence of classes in the sense above criticised.

Now we can also see that diagnose (i) (at the beginning of § 40) is beside the point. Besides, it is, I think, a curious enough thought that a collection may be illegitimate in any way whatsoever. Surely we can consider and class together objects of whatever kind and category. (The only illegitimate collection, so to speak, is an infinite collection, but this is no collection at all.) The only impulse to the opposite view arises, I think, out of the presupposition that there are classes as objectively existing things.

I should like to conclude this long discussion with a few remarks concerning the status of classes, if there were such things. It is clear, I think, that they would admit as members objects of whatever category and further that even one and the same class could consist of a mixed and categorially heterogeneous 'population'. Now even a simple survey would show, I conceive, that classes conceived as objective items, would not fit to any of the five categories acknowledged till now:

-- a class is not a fact; a fact is a whole, a class simply a collection^I.

- I. It is indeed interesting to contrast the organic, as it were, complexity of a fact with the mechanical juxtaposition or conglomeration of a class. Even the expression 'mechanical juxtaposition' ascribes to the class more than it really has and deserves; for a mechanical juxtaposition may be something as objective as spatiotemporal proximity or mechanical conglomeration, whereas a class is just a co-conception on the part of some thinking mind of any objects whatsoever.

- a class is not a determinant; it cannot be exemplified or not exemplified¹.
- a class is not an instance; it does not inhere in a concrete thing.
- a class most clearly is not a tie; it does not connect anything.
- a class seems not to be a concrete thing even if it consists exclusively of concrete things as members. For concrete things are characterised in themselves, classes are not characterised in themselves, they are characterised only in respect of their members.

To be sure, the above brief remarks require supplementation and possibly qualification. Nevertheless they indicate, I think, that classes would fall under the category of class---a sixth category of objects. Denying objective existence to classes, we also comply to the principle of intellectual parsimony.

41/ The analysis in § 40 is, I submit, multifariously useful for our discussions in the present essay.

i/ It shows the fallacy committed in an argument commi- like the one in § 39.

ii/ It makes clear (in so far as it succeeds to do so) that we have not to postulate classes among objects, as constituting a separate category of them.

iii/ It establishes the point of § 36-38 concerning the distinction both in respect of subject matter and general methodology between philosophy (here metaphysics) and science (here mathematics). For even if the correctness of the above analysis is not admitted (and that analysis is not propounded as an adequate one, at any rate), still the acceptance of its philosophical character suffices for the establishment of the above distinction. For if it is

17 Of course we have also suggested construal of classes as a certain relation, hence as a kind of determinant. But then this is not the normal and ordinary conception of a class, which we are assuming in the above remarks, but a technical substitute for it.

2. A class conceived subjectively is indeed a sort of mental grouping. But the tie of which we speak above is an ontological connector.

granted that the analysis is philosophical, and given its sharp contrast with anything that could reasonably count as a mathematical disquisition about classes (in the theory of classes), it clearly exhibits our point. It also provides the guiding lines for an analogous treatment of similar cases.

iv/ Finally the above analysis has yet another bearing on our discussion of the notion of infinity. For it shows, I think, the root of the connection between the notions of infinity and class employed in 39. For it must be noted that I have not objected directly to such a connection; only by restricting classes to their proper place and construing them, to put it roughly, as mental constructions, I have shown the precise nature of that connection. And the connection is that of course we can proceed in the way indicated in 39 ad infinitum, i.e. ad indefinitum. In other words, certainly we can conceive of the class of n objects as a $n+1^{\text{th}}$ object forming a new class with the n ones; and again we can conceive of the newly constructed class as the $n+2^{\text{th}}$ member of a class consisting of it, the other class and the n initial objects, and so on. There is no limit to this procedure and this is what we mean by calling it 'infinite'. Therefore the infinity of which we here need is really again only a potential infinity, not an actual one; it is just the possibility of continuing in our construction of such a series of classes one more step further, irrespective of the step already achieved. And we tend to mistake this infinity for an (impossible)actual one, only under the presupposition that classes really exist as independent objects, and are there to be discovered, so to speak, or at any rate to be apprehended. And our analysis of 40, if correct (at least in principle), discredits exactly this supposition.

42/ We find ourselves, then, being still justified in not seeing how one can escape from the privative character of the notion of infinity. Summarising: infinity is just the lack, the privation of quantitative determination; nothing can exist indeterminately, therefore nothing infinite actually exists.

Two remarks, one explanatory and one qualifying and supplementing, are perhaps at place here.

I/ We speak of the privative nature of the notion of infinity and not of its negative nature simpliciter. For infinity is the absence of quantitative determination to that which is, by its nature, susceptible of such determination^I --- that is, to a quantity or something that has quantity. A quality or a sound (even a point) are not quantitatively determined per se (if they are at all, they are accidentally, in virtue of something else); but this does not mean, of course, that they are infinite; nor are we committed to the view that they are infinite in emphasising the non-positive character of the concept.

ii/ WE feel that infinity is not simply indefiniteness. But even if this is so, it is so not because infinity moves away or diverges, so to speak, from indefiniteness towards determination, but because it is just a kind of indefiniteness. But someone may object here. He may claim that 'infinite' vulgarly connotes the prodigiously large or great in magnitude (if the quantities in view are continuous) or the 'enormously many' or great in number (in case the quantities are discrete). And, he may argue that consequently, we must take account of this shade of its meaning. And a rational way of accommodating the suggestion involved in the vulgar meaning of the term is by saying that the infinite is nondeterminate (he may even concede to us that it is indeterminate) in a special sense, and not by having just a kind of indefiniteness.

The objector may develop his attack as follows:

"You have already accepted implicitly two senses of quantitatively indeterminateness or indefiniteness;

(a) In the one, the expression signifying simply the absence of quantitative determination from entities or objects not admitting, according to their nature, it. And everything which

I. Strictly speaking we may perhaps distinguish two senses of privation:

i/ the absence of a (positive) character or determinant from an entity susceptible of its presence, in the sense that the essence of the entity of question does not preclude that presence.

ii/ the absence of a (positive) determinant from an entity which, according to its essential nature, calls for its presence, in the sense that its absence is or indicates a defective development of that nature, an incomplete realisation of it in the corresponding respect.

But for our purposes here, we shall have not an occasion, I believe, to call attention to this distinction.

is not a quantity or has per se quantity is naturally unsusceptible of quantitative determination.

(b) In a second sense, the expression signifies the absence of quantitative determination from what is, according to its nature, susceptible of it and calls for it. Now admittedly nothing so susceptible can exist lacking this kind of determination^I. This

I. And this constitutes a major difference from ordinary cases of privation. For normally nothing hinders something from 'having a privation' (i.e. not having the corresponding positive nature or character) or, if one permits the expression, from being "privated" --- and this means that the question-may thing in question may exist 'together' with the privation. That is, for ordinary cases of determinants, if P-ness is such a (positive, needless to say) determinant, then there is no impossibility involved in there being a thing which is not characterised by P-ness even if it is susceptible of such characterisation, or even if it would have to be so characterised, according to its nature. Not so with our present case: for for some determinable determinants (such as are situated very 'high' in the determinable-determinate 'ladders'), if something is really susceptible of their characterisation (or much more, if its nature calls for such characterisation --- and here, it is significant that we cannot easily differentiate between the two senses of 'privative') then it must be so characterised, and hence it must also be characterised by determinate determinants under the said determinable one. So, if a kind of things are susceptible of colour-determinations, then no one thing of such a kind can exist without being coloured and, therefore, without having one or other of the determinates under colour. Similarly here: if a kind of entities are susceptible of quantitative determination, then no entity of this kind can exist lacking such determination.

On the basis of the above note, we have to add a third sense of 'privation' to the two senses recognised in note I to p. 49, the following:

iii/ the absence of a (positive) determinant (usually 'highly' determinable) from a kind of entities, whose nature requires its presence, though, perhaps, it does so not exactly in the sense in which a constituent (or constitutive element) of the nature is required by that nature.

It is this sense of privation (iii) which 'annihilates' its "purported" subject-entity---so to speak.

negative result is so far justified, only it needs supplementation provided by the recognition of a further case (c).

(c) We have here something susceptible of quantitative determination(a quantity or something having quantity), and yet it lacks any such determination without being ipso facto rendered non-existent, only because its privation is due to the fact that, so to speak it transcends, it exceeds all measure. We must consequently acknowledge this third case (where the subject, though susceptible of it, lacks nonetheless all determination, transcending it) and by so doing accommodate the valid element in the figurative vulgar conception of the infinite as that which is prodigiously great(in magnitude or number)".

Notice that this objector in (c) is not saying that the infinite is still determined, though not finitely(in the ordinary way) determined; such a claim involves the idea of infinite as a kind of quantitative determination, which I have tried to show that it is incongruous.^I

So the objector. But now to the point, even if accommodation is really needed for the last mentioned idea or conception, the above will not do. For the new idea is the one of a quantity not being quantitatively determinate because no quantitative determination can comprehend it. And this seems to be an equally incongruous idea. For any quantity must be one or another determinate quantity, must have one or another determination down to the absolutely determinate determinations; quantity must be one or another determinate quantity. And this shows the illegitimacy of the idea.

But in spite of this I do think that the objector points to the right direction; for it seems that, incongruous or not, the above formulated idea is what can be extracted from the ordinary conception of infinity(of course the formulation is not ordinary, but philosophical; the idea is ordinary and not philosophical).

Therefore finally we have the following definition of "infinity":

I. The new idea has a marked affinity with certain ways of thinking of the Neoplatonists(A structural affinity),

Infinity is the privation of quantitative determination (i.e. the absence of it from something susceptible of quantitative determination (ie. a quantity or something having quantity)) due to the quantitative 'super-abundance' of the subject which overpowers any restrictive quantitative determination.

Needless to emphasise that the idea of infinity conceived either in the above way with the addition, or simply as the privation of quantitative determination, is fundamentally incongruous, according to the above analyses. Hence nothing infinite exists actually or even potentially, in the ordinary and proper sense in which if so-and-so is potentially, then it is possible that so-and-so, and, hence, it is possible that it should actually be the case that so-and-so.

P

43/ Let us try to gain a somehow general view of what we have done till now in disentangling issues regarding the notion of infinity. Firstly we have explained (or rather explained away) infinity as applied to operations, let us call the notion of infinity involved here 'the notion of operational infinity'. The task of explanation was discharged mainly in 77 18-25. This operational infinity, as explained, proved to be ontologically noncommittal. But there, secondly, a kind of operation was brought under notice, which precisely through its operational infinity seemed to require ontological infinity. (77 26-29). In 77 30 we have seen the thing from its other end, denying that such operations which do require ontological infinity are really (operationally) infinite. And to substantiate the reason of this claim we have thirdly embarked on an investigation of the notion of ontological infinity (the one applying to entities and not simply to operations¹), in 77 31-43. There we have tried to show that the notion of

I.

One may say: "Well, operations are entities or objects, too". Good. And then equally well these cannot be ontologically infinite (either actually or potentially). What they can be is operationally infinite---the quite special sense in which an operation qua operation (and not qua entity) may be 'infinite' ('capable' of continuation ad infinitum).

ontological infinity^I is incongruous. And hence, in so far as we have succeeded in showing this, we have also justified the "reverse point of view" of 30.

44/ So, concluding: we have the definition in 42 above of the notion of ontological (non operational)infinity--let us call the definition 'D'. We may also formulate the following definition of the notion of operational infinity:

An infinite series of operations is an interminable series of successive applications of a given operation, in the sense that whichever step has been reached in the successive application of the said operation, a further step can be taken by applying the operation once again.

Or, better, we may explain that:

(D_O) An operation can be applied an infinite "number" of times, or can be applied ad infinitum, if it can be applied once more, having been already applied any number of times, irrespective, that is, of the number of times of its application till a certain given point.

It is to be noted that in (D_O) the first occurrence of 'number' is put in inverted commas (whereas the second is left free of them) to mark its, strictly speaking, impropriety (or, what amounts to the same, its propriety if meant in another than its primary sense).

Secondly, it must also be emphasised, that the 'can' of (D_O) is also an improper (or secondary) 'can'---for it does not mean possibility of actualisation of a compete and completed infinite series of applications of the said operation, but only possibility of proceeding one step further from any step actually (or thought of as actually) achieved and hence capable of being achieved (in reality or in thought) in the primary sense of 'capable' (and the relevant one).

I.

I call it so, to distinguish it from the special operational infinity, the indefinite continuation of a series of operations.

45/ Now obviously we are driving at this point: whether an operation can be applied on a certain subject, and, if it can, whether it can be applied ad infinitum, are questions whose answer depend on the nature of the operation and of the subject-matter involved. But however this may be, what is impossible anyway, if our discussion is correct, is the anal. actual or even possible existence of anything infinite, in the defined "ontological" sense¹.

46/ After the above clarifications, we can, I suppose, approach now, somehow more fruitfully, our problem of the validity of the regressus ad infinitum form of argument.

In such a form of argument, upon a certain claim as foundation (upon the initial move m_0 of 413) an infinite series of either conditions or consequents is constructed. This construction can be viewed for our purposes as the successive application of a certain operation specifiable from the conditions of the case. Hence, in such a form of argument, operational infinity is necessarily involved, which further it allegedly implies an ontological infinity either in the direction of conditions or in that of consequents.

Now in the form of strict finitism which I have tried to defend, it is immaterial whether the ontologically infinite allegedly implied in a regressus ad infinitum is established in the field of conditions rather than consequents or vice versa, (Such a consideration would indeed be material for the strict finitism envisaged en passant in 418); for (ontological)infinity itself is the vicious element. Hence the regressus ad infinitum, if correct, suffices to overthrow the claim on which it is based. The only way to escape this, is by establishing its fallaciousness. And one important point at which the fallacy may inhere is the transition from operational to ontological infinity.

47/ We may, perhaps, try to clarify these remarks through the comparison of two examples.

I. This is indeed evident, provided that we accept definition (D) and the principle of determinateness.

Example I. Claim(initial move): any rectilinear line-segment can be divided into two equal rectilinear line-segments.

Regressus ad infinitum: suppose a certain rectilinear segment is given. It can be subdivided into two equal such segments, each one of which can be further subdivided, and so on ad infinitum.

Conclusion: according to the above regressus, any given rectilinear segment consists of an infinite number of rectilinear segments. But this is impossible (according to the strict finitism here examined and expatively propounded). Hence the claim is false; there are, after all, indivisible lines¹.

Example II. Claim: any voluntary act is "caused" (according to efficient causality) by a volition, which is a voluntary act of the will.

Regressus ad infinitum: since the volition is a voluntary act, it must be caused according to the stated principle by another act of the will which also has to be voluntary if the said first volition is to be voluntary. And the same holds reasoning applying also to this second (order) volition, we face a regressus ad infinitum.

Conclusion: but then there must be an infinite number of acts necessarily connected with any given act. And this is impossible. Hence the above claim is untenable; it has to be modified² or abandoned.

Now of course there are various ways of meeting arguments like the ones expounded in the two examples above. But any of these ways must make place, I conceive, to the clearly enough felt difference

1. The claim amounts of course, to a rejection of the doctrine of indivisible lines (Compare with the problem of Έπιχονία in ancient philosophy).

2. For example by qualifying it to read as follows: "any voluntary act is caused by a volition, which is an absolutely special, or rather unique kind of act of the will, its uniqueness accounting for the distinction voluntary/involuntary not being applicable to it". Or, alternatively, we may say that it is unique in that it voluntary in a primary sense, on which the secondary sense of the voluntariness of ordinary acts depends.

between these two arguments, a difference which, *prima facie*, goes far enough to render the first invalid and the second valid.

And our way of handling the situation conforms with this vague intuition and pinpoints the difference. For in example I only operational infinity can be validly extracted from the initial claim. For a subsegment of the initial segment ensuing upon the n^{th} subdivision of someone of the segments produced upon the $(n+1)^{\text{th}}$ division is brought into existence by the corresponding subdivision in the sense that it acquires actual existence (and separable existence) just as a result of the actual corresponding division. In this way it also becomes an (actual) part of the initial segment just in virtue of that actual dividing. Hence the initial segment has always a finite number of (actual) parts---this number being determined from the number of the actually made (in thought or in reality) divisions, which number cannot but be finite as we have explained, since the infinity pertaining to the operation of subdivision is simply an operational infinity consisting in the possibility of further subdividing any segment taken as a result of previous subdivisions.

(Clearly also the answer to the question " How many potential (as distinct from actual) parts does a given segment have?" is indefinitely many, not infinitely many---or rather it is the latter in so far as it means (being conceived operationally) the former).

So that what is concluded in the conclusion of the example I does not strictly follow from the regressus instituted upon the initial claim, or, in other words, the transition from the operational to the ontological infinity is not valid in this case. Hence, no need arises, in so far as the said argument goes, of modifying the initial claim.

On the contrary, in the case of example II, the infinity involved cannot be restricted only to the operational kind. For we cannot possibly maintain that the cause is constituted or brought into existence as a result of the application of the operation asking-for-the-cause-of-voluntary-actions. Indeed, the issue is so much removed from any consideration about our own involvement in it, so that any mention of 'operation' here sounds unnatural. Anyway, it is clear I think, that it is ontological infinity which the argument (and the initial claim) of example II requires, and hence the corresponding conclusion is, in this respect, correct.

48/ And now we may, perhaps, be able to tie up one last, but essential, loose end.

In 712 we were faced with a regressus ad infinitum argument against the positing of tie as a further ultimate category of entities or objects.

An immediate inference from the above given analyses of the notion(s) of infinity, is to the conclusion that, for a strict finitist, when faced with a regressus ad infinitum, there is no escape through an attempt at distinguishing between vicious and virtuous regressi (for example by recourse to the contrast of a vicious series of conditions against the not necessarily vicious series of consequents). It is true that we have tried to provide him with another principle of estimating such arguments---the distinction between operational and ontological infinity.

But since our claim in positing the category of tie is clearly metaphysical, and is not content to be an account of what we normally or ordinarily (though perhaps implicitly whatever this may mean) say or think just in so far as we say or think it^I, the above distinction and the corresponding principle of estimating regressi is of no avail to us.

Still, the argument of 712 is indeed fallacious, I should think, in another respect. And this is the illegitimacy of proceeding further than the first step.

We have said that the connection of concrete things to their determinants is "effected" by a tie. But for it, there could not be a characterisation or determination of the concrete thing by the abstract thing. A relation, conceived as an abstract thing and not qualifying concrete things, cannot afford to connect concrete things by itself; just as a quality, conceived again as an abstract thing and not as qualifying a concrete thing, cannot afford to qualify concrete things by itself; and generally a determinant, conceived again as an abstract thing and not as determining concrete things, cannot manage to determine things by itself. A tie is needed. But then the tie just connects items of the two different categories of concrete and abstract things just in so far as it is a tie (just qua tie) and not by means

I. For, after all, we may feel inclined to say: it is with what we philosophically say or think that philosophers are concerned with, and even this in so far as it is claimed to be so objectively in reality.

or via or on account of something else; this is precisely its function, or rather its essence and nature (though the function of a tie seems to be its essence); the function of a tie is simply to tie what we have said; and it performs this function qua tie.

It may be objected that, then, neither the first step is really necessary. For why not say simply that a relation relates concrete things qua relation, that a quality qualifies concrete things as quality, and, generally, that a determinant determines things qua determinant---and remain content with this?

I dare say that the proper answer to such an objection is just a negation of what it alleges to suffice. For self-evidently, I conceive, we can distinguish in the relation of certain given concrete things (the way they are related) the relation as an abstract thing from the relating (the "relationing" so to speak) of the given concrete things by that relation, as well as we can distinguish in a determinant as determining certain concrete things, the determinant itself from the determining of the concrete things by the determinant, and this determining¹ in general, considered in abstracto from both the determined and the determinant is what we signify by 'tie'.²

It is in such a way, I think, in which the argument in Q 12 must be met.

49/ One final word: the account of (ontological) infinity presented in this essay, has to pass three further severe tests before being established on a firmer basis than that of an explorative tool: the three severe tests I have in mind are the alleged infinity of (natural) numbers, of space, and (apparently most important and difficult for an adequate handling) of time. But of these tests, in a subsequent essay.

1. Clearly 'determining' is ambiguous; for it may mean either the determining in abstracto both of what determines and what is determined, or that which determines. We keep the technical term 'determinant' to signify the later sense and we apply 'determining' in the first one.
2. One may profitably connect this discussion with Frege's view about the necessity of an "unsaturated" element in thought and hence, presumably, in reality. Only he acknowledges concepts as such "unsaturated" elements. According to the view in the text above, in any such Fregean "unsaturated" element one can distinguish a "saturated" and a completely and absolutely "unsaturated" component: the first is the abstract thing, the second the tie.

APOSTOLOS L. PERRIS

SOME REMARKS ON THE
NOTION OF INFINITY

AND THE
REGRESSUS AD INFINITUM
IN METAPHYSICS

[WORKING UNCORRECTED COPY]

(Sometime in 1973)