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On Indivisible Lines and Temporal Atoms

The Problem of Space-Time Continuum in Ancient Greek Thought

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Aristotle surely put his finger on the real core of the trouble about ἄτομα μεγέθη (μέγεθος being something almost ex definitione συνεχές) in that most important first book of Physics, known in the antiquity not without reason as Περὶ ἀρχῶν. Cf. I.3.187a1 – 3 and sqq. : ἔνιοι δ' ἐνέδοσαν τοῖς λόγοις ἀμφοτέροις, τῷ μὲν ὅτι πάντα ἕν, εἰ τὸ ὂν ἕν σημαίνει, ὅτι ἔστι τὸ μὴ ὄν, τῷ δὲ ἐκ τῆς διχοτομίας, ἄτομα ποιήσαντες μεγέθη. What Aristotle really believed about such intellectual weakness that causes oppressed doctrines, is rendered explicit in the tract about Indivisible Lines, *περὶ ἀτόμων γραμμῶν*, 969b3-6; v. *infra* n. 25.

Two Eleatic arguments were considered so powerful that some people thought of them as unassailable – and they succumbed to their force. These two arguments are (1) the inference of the oneness of being from the nonexistence of non-being (given that the non-being does not exist being is not susceptible of multiplicity, since every multiplicity implies otherness of its members one to another, and otherness postulates non-being: what is other from another, *is not* that other thing), and (2) the argument from dichotomy that provides the starting point of my investigation here. Those that felt the overwhelming power of these arguments were driven (Aristotle explains) to negate the significant premises upon which the Eleatic arguments were based, which meant that they were forced to accept (a) the existence of non-being and (b) the existence of indivisible lines respectively.

I have discussed the first point with reference to Parmenides and Plato elsewhere. Now we are concerned with the second point, about διχοτομία. As it is made clear from Simplicius' commentary *ad loc.* (and the other ancient comments as e.g. edited conveniently by Brandis in the fourth volume of the Berlin Aristoteles), the matter stands thus. Zeno's ultimate principle can be succinctly formulated as follows: *The real cannot be contradictory.* If therefore a putative reality is shown to be contradictory, it follows that it cannot be (really) real. [Precisely the same form of argument is used universally, thoroughly and systematically in particular by Bradley]. Thus, he tried to deduce various contradictions about what people usually hold as realities – the many things of the empirically perceived World.

The particular argument which is connected with the matter at stake is this: Take any *one* empirically perceived, physical, concrete thing; it necessarily has *μέγεθος* (it occupies some portion of the space; it is extended); now any μέγεθος can be divided into parts; therefore the *one* thing is also *many*, that is as many, as its parts

are – here we have a first contradiction. But Zeno seems to have proceeded further; it is not only that any *one* physical thing is also *many*; it is also that we can never say exactly how *many* things it is. For each of its parts can also be divided further on *indefinitely* (since μέγεθος is συνεχές). Therefore it is not only that taking some things which *appear* to be one, we always discover that they really are *many*; that would have been innocuous and harmless enough (that is, if one was prepared to accept an atomistic interpretation of being starting from Melissean arguments); but the stronger point is that with *every conceivable* physical thing *it is both one and (indeterminately) many*. And this was held by Zeno to be both an inescapable and palpable contradiction. Plus the fact that indefiniteness does not exist, and infinity is another name for indeterminacy.

This decomposition of physical, extended, concrete being as such seems to render evanescent its reality. For Reason demands some end to that process; Reason demands a unity; or, in the absence of unity, a definite, determinate (even if unknown and even unknowable) multiplicity, which, precisely qua determinate, reposes on ultimate absolute (indivisible) unities. [It is such an annihilation of material being in the hands of the "idealists" also which is meant by Plato in the *Sophist*, 246b-c].

What can be said about this powerful form of argumentation? The answers given to it in antiquity fall under the following types. (I do not count the full acceptance of the conclusion – the Eleatic point. I am here interested in the proposed resolutions of the force of the argument).

I) The first kind of response consists in accepting the *force* and validity of the argument (ἐνδίδω τῷ λόγῳ, as Aristotle puts it poignantly); but to deny the conclusion by negating one of the *premises* of the argument, namely that one can proceed with the division of μέγεθος indefinitely. This type of theory maintains that, if it is for material things to exist, then their divisibility must be limited; there must be something indivisible in the end.

This type of view is further subdivided, logically and historically, into:

a) The thesis of those who posit indivisible *bodies*, minimal material things, in the end of the analysis, like the Atomists. The important thing is that they *need not* refuse to maintain that extension qua extension is *indefinitely* divisible; it suffices if there is a limit in the decomposition of physical things, if there are certain ultimate minuscule bodies indivisible *physically* διά σκληρότητα [1]. And this exactly emerges to have been the position of the Atomists; for them each atom was just like the Eleatic being, it was πάντῃ ὄν, and not πῇ ὄν – only that they interpreted it materialistically:

πάντη ὄν was the *πλῆρες*, that which is without any ῥωγμή as it were in which their not being, i.e. the *κενόν*, may enter. Think of the absolute homogeneity and isotropicality of Being in Parmenides. You see here precisely the same *model* interpted metaphysically by the Eleats and materıalistically and physically by the Atomists.

The Atomic position is not particularly cogent. Already Aristotle had demolished it – that it reappeared in Epicurean philosophy is one of the many retrograde movements in the history of Ideas. In short the demolition works like this: take one of the *ἄτομα σώματα*. It is extended (as material, physical being, i.e. as body) and hence it occupies space; now the space, as pure extension, is indefinitely divisible; take then the place occupied by an atom and divide it, say, into two parts; you cannot say that the *whole* atom is on *one* of the two subspaces; it must occupy both of them *partly* being in the one, *partly* in the other; hence the atom has parts; hence it is divisible – whether it can be divided physically or not, it is divisible metaphysically. The indefinite divisibility of space as extension introduces metaphysical faultlines everywhere within the most solid physical existence, and hence breaks it apart insofar as its metaphysical foundation of existence, and thus its true reality is concerned. Quod erat demonstrandum.

b) The stronger, "metaphysical" version of the (I) type of response to Zeno's argument would be to claim that even the μεγέθη as such are *not* indefinitely divisible. This view is ascribed to *Xenocrates*, who postulated ἀτόμους γραμμάς [γραμμαί are the primary μεγέθη as it were, being in one dimension what ἐπιφάνειαι are in two and σώματα [2] are in three].

Xenocrates' position is untenable as a theory about μεγέθη as such, as a mathematical theory [3] – and as such it was apparently proposed (unless one (mis)interpreted him in the Neoplatonic way, v. end of n. 1). Aristotle argued against his contemporaneous Head of the Academy without mentioning him, ex professo in the initial chapters of Book Z of Physics [4]. The substance of the reason against is given in a short passage right at the beginning of the Book (231a18 – b18), in the typically disorderly Aristotelian way. (What follows is supplementary, in a sense which is made most clear by 231b18-20).

My constructive reconstruction of Aristotle's not so "deep" reasons against the Xenocratean theory is, in brief, this: Suppose a line is divisible into a (definite) number of ἄτομοι γραμμαί, and is composed out of them. *An ἄτομος γραμμή must be ἀμερής* – otherwise it would be *divisible* [5]. Take one of these. And ask how it is combined to its neighbouring one so as to form the entire given line (since there is a

definite, however large, number of them, there must be a *next* ἄτομος γραμμή to each one); the possible answers to this question are the following, each leading to an absurd consequence according to Aristotle:

1) There is no contact between two consecutive ἄτομαι γραμμαί. But then the line composed out of them cannot be συνεχές, for in a συνεχές magnitude, between any two parts of it not in contact there is always another part of it. And it was *assumed* that lines (except ἄτομοι lines) *are* συνεχεῖς [6].

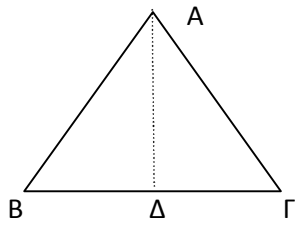
2) The one contacts the other [7]. But to contact is to have the respective πέρατα at the same place; now if something has πέρας, it must have something which περατοῦται by the πέρας – and hence it must be composite.

One sees, these are at most verbal reasons [8]. The Xenocratean position is meant to destroy, not to uphold, the continuum in lines and magnitudes. And as to the second, and apparently more forceful, objection, it is too easy to expatiate on the differentiation between the interior and the limit of a limited entity, and to reckon varying distances from the interior to the limit, and hence to create superimposed and overlapping parts within the bounded atomic line; but lost in this easy-going train of thoughts we are likely to forget that all such reasoning does not apply to the unit of magnitudes, any more than analogous overextensions of vacuous thinking apply to the numerical one in its relationship to the different numbers. The indivisible elementary line is the unit of magnitudes and is always and everywhere of the same "length", just in the formal way that all ones, as units of number, are of the same "number". The boundary that makes of an extension the unit of magnitudes is as appropriate a way of thinking as the elemental limitation that creates out of multiplicity the unit of number. The oddity of one as a number is exactly reflected in the awkwardness of an atomic line constituted as of such an extension as to be the unit of length and all extension.

Admittedly this resolution of the initial Aristotelian counterarguments to the thesis of the indivisible lines goes a long way toards the arithmetization of geometry. And as this interpretation would appear to be counterintuitive, one may appreciate the reason why I emphasized that we must move to the realm of ordinary mathematics in order to be able to efficiently combat the view. And here the Aristotelian tract Περὶ ἀτόμων γραμμῶς is valuable, for it proposes to refute that there are ἄτομοι γραμμαί simpliciter – without restrictive and convenient conditions (like (1) above). Reasons against the Xenocratean doctrine are given in the middle portion of the tractate, 969b26-971a3; and all of them, directly or indirectly, in one way or another, move on the mathematical level.

Now I can only register here my view of them, and exemplify my thinking without much ado. Many of the Aristotelian objections and counterarguments can be answered by the adherent of the ἄτομοι γραμμαί quite easily; others require a special inquiry, which I shall not pursue in detail here. I shall adduce two examples, one of each category, to explain roughly what I mean.

- (i) Easily answerable objection. Take three elementary lines and form an ἰσόπλευρον τρίγωνον with them as sides, so



Now draw the perpendicular from A to ΒΓ, i.e. ΑΔ. It is a geometrical truth that ΑΔ < AB and that ΒΔ is half the ΒΓ. But AB and ΒΓ are the elementary, ἀμερεῖς lines, and nothing can be shorter, or half, of them.

The objection can be countered thus: the triangle formed by the elementary lines is an elementary triangle; i.e. it presents itself as a unit, nothing can "happen" in the inside of it – there is no real *inside*, etc.

- (ii) An apparently unanswerable (mathematically) objection. Every line (even granting the exception of an ἄτομος line) is divisible into two equal parts. This is presupposed in all geometry – a universal, indisputable presupposition. But now take a line composed of an odd (περιττός) number of elementary lines, say 9. Obviously it cannot be divided into two *equal* parts for then one elementary line would have to be halved, 4 ½ + 4 ½ = 9.

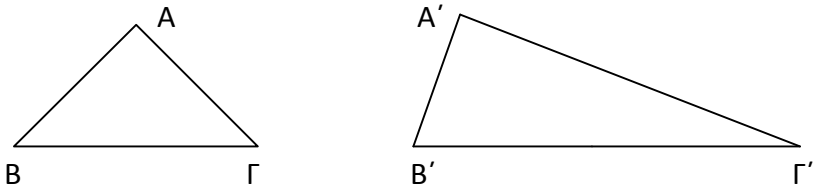
I cannot see how this can be answered. Nor what a proper mathematical resolution of the difficulty might be. Unless, that is, one takes the bull by the horns and denies the common geometrical rule about the possibility of dividing every line into two equal halves. Remember that in 5th century mathematics were in the making. The question encountered then were foundational issues – and indeed such pursued before the raising, let alone the completion, of the edifice in its imposing Euclidean form. One may compare an indirect parallel in the mathematics of the musical theory. Starting from the consonant intervals of the perfect fourth (4:3) and the major fifth (3:2), the tone is deduced as the interveal by which the fifth is acuter than the fourth with a common base, and this is equivalent to the values 9:8 (= 3:2::4:3). Now the fourth is acuter than two consecutive tones starting from the same sound base by

a semitone, whose interval (S) is accordingly defined by the equation (9:8)x(9:8)x(S)=4:3. And thus the value of the natural (and *not* the well-tempered) semitone is 256:243. Which is less than the mathematical "length" of half a tone, since the natural semitone equals ~1.05350 and the "well-tempered" semitone s defined as the exact half of a tone (and thus determined by the equation sxs=9:8, with the value s=3:√8= ~1.06066). The harmony of nature here detests exact halving of the tone and prefers strange ratios instead. In a Pythagorean setting as that of the Old Platonic Academy, the analogy would be most telling. And thus the negation of the general geometrical truth that any line can be divided into two equal segments might be adopted as a corollary of the foundational principles of absolute geometry.

Further on the issue, and before finishing this (b) section of the (I) type, what, we may ask, is Plato's position as to the matter? But previous to that, we must always look for the Pythagorean views on any matter concerned. Now Aristotle reports a Pythagorean doctrine which, I think, lies at the root of all the I-type developments. See, mainly, *Metaphysica*, 1080b16-21 (and cf. 1083b8-18). They conceived numbers as the very constitutive substance of physical things (not as a mere paradeigm) – and since numbers are composed from units, they onceived of the *numerical* units as somehow *extended*, in order to account for the extension of physical things. But, Aristotle adds, they could not explain how the first *One*, the first material unit, was extended, in the first place. You see here the primitive core of all subsequent troubles, from which there sprang both the (a) and (b) types of development.

For Plato we have the valuable passage *Metaphysica*, 992a19-23. There may be some disorder in the text (though on second thoughts I do not think so) – but in any case Alexander's interpretation (the same as Asclepius') is correct. Plato thought that the existence of points was nothing more than a geometrical "working hypothesiws" as it were, or rather, worse than that, just an expedient "trick" without any reality to answer to it [9]. He of course admitted that there is a principle of lines – but such a principle was not a point, but what in many cases he called "ἄτομος γραμμή".

This squares perfectly with what we are tought in *Timaeus* [11]. There are certain (two, in fact) elementary triangles out of which the physical elements are constructed, and therefore all the material world. These two fundamental kinds of triangle are the following:



$\hat{A} = \hat{A'} = 90°$(ὀρθὴ γωνία), AB = AΓ (ἰσοσκελές τρίγωνον); (B΄Γ΄) = 2(A΄B΄), in which case $\hat{A'B'\Gamma'} = 30°$.

Now, although this doctrine agrees as to the *general* impact with the doctrine of ἄτομοι γραμμαί, yet if one tries to trace in *detail* the correspondence serious difficulties arise, whose solution sheds clear light in the *non-mathematical* significance of the doctrines in Plato's eyes [12]. For suppose AB = AΓ = A'B' = α, and also suppose that these are the elementary, ἄτομοι, lines. Now obviously BΓ, A'Γ' and B'Γ' are greater than α, the ἄτομος γραμμή, but not twice as α (except B'Γ') – therefore they must be one α *and a part of it*, which is impossible, α being the ἄτομος γραμμή. Further, it can be shown that (BΓ) and (A'Γ') are *incommensurable* with α.

What is to be said as to this impasse, similar to the one we encountered above, – which is irritating because you feel that it must amount to nothing at bottom! This is my solution at this time: The ἄτομος γραμμή is not a mathematical line like all others – it is the principle of lines. The difference between the two above drawn triangles does not consist in the different length of their respective sides but in the fact that in each case a different *form* has seized *three elementary lines*, all six of them derived from the principle of all lines – the ἄτομος γραμμή. In this way the elementary triangles cannot be broken, the fundamental structure of this *όσμος* would then collapse, pulling with it the entire physical World, there would be a relapse into that absolute disorder which (metaphysically if not temporally) preexisted and preceded the imposition of order. That very imposition of order is effected by certain "injections" of order at the foundations of the fabric of the World. Such injections of order are the indivisibility of AB, AΓ, BΓ, A'B', A'Γ', B'Γ' – and in this consists their being images, as it were, of the Principle (Ἄτομος Γραμμή), not in that they are equal in length in a geometrical sense. A'Γ' is incommensurable with A'B' – all right, but it is made so by the requirements of the *form* which *grasps* three images of the ἄτομος γραμμή and makes out of them a definite kind of σκαληνὸν τρίγωνον. A'Γ' obeys the requirements of the reigning form and is *made* what, according to that form, it should be. But as a side of an elementary triangle it is just an image of *the* ἄτομος γραμμή, and therefore *an* ἄτομος γραμμή itself, as A'B' ex hypothesi was.

This much as to my guiding idea in so far as the interpretation of the Platonic position will have to be [13]. Xenocrates, by contrast, would model geometry itself (and mathematics more generally) on a different pattern adapted to the foundation provided by his principles of being, as indicated above.

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II) Aristotle brings in the new moment to this "problematique" by his usual panacea – the distinction δυνάμει - ἐνεργείᾳ! See, e.g., the very instructive passage in *Physics* Book A, 185b25-186a3. He believed that so long as a συνεχές was actually undivi*ded*, it was actually one, and only *potentially* many, since, according to its essential character, it is only divisi*ble*, and this, he thought rather superficially, is no contradiction – Zeno is therefore silenced. This in effect is to treat mathematical extension as physical cohesion, and clearly will not do in an Academic context.

Afterwards in the ancient tradition, the Aristotelian view was generally accepted. Only, in Neo-Platonism the emphasis lies in the right direction: one accepts Zeno's argument in toto; one agrees that therefore physical reality cannot be *real* reality. But now between absolute reality of Being and Nothingness we have learnt to posit various grades of inferior reality. So, arguments like Zeno's are now taken to prove the ὑφειμένη reality of material existence (and of extension, as necessary condition and structure of material existence), rather than its total unreality. But at bottom the idea is "Aristotelian": if physical things are ἕν and πολλά, they are so ἄλλως καὶ ἄλλως. But they enjoy enough of unity not to be submerged into absolute Nothing; and still they are vitiated by enough of chaotic multiplicity, not to be able to keep on the level of true being. As everywhere, Neo-Platonism endeavours to synthesize Platonism and Aristotelianism, with differing results and success - in some cases there is compromise, in others syncretism, while in many one reaches the level of genuine harmonization. There is much in these attempts that amply repay close investigation, as will be shown by example in a moment. And in any case, their "fullness" is highly commendable.

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Before turning to Damascius' very singular doctrine, let it be observed that whatever was said above, was concerned with extension and magnitude - spatial. Now this is one of the two great categories of fundamental συνεχῆ; the other is temporal duration on the one hand and *movement* on the other – movement and change involving necessarily and essentially time whether or not they also necessarily entail change in *place* – which is true in the case of locomotion and, for Aristotle, for any other change as well which, for him, involves necessarily locomotion as the primary change.

Aristotle emphasized the absolute correspondence between the *extensional* and *durational* categories of συνεχές. For instance see the argument in *Physics*, Z, 233a13-b15 (esp. 233a21-34) against Zeno's contention that if there is an infinite number of divisions between any two points A and B, and if a moving body transverses the

distance from A to B in finite time, then it would be possible to go through an infinite number of places (make an infinite number of steps) in a finite time – which is absurd. Aristotle in answering that *in the sense* in which the spatial distance from A to B includes an infinite number of places, it is also true that the time taken by the moving body to transverse that distance involves an infinite number of time-divisions (and "steps"); and in the sense that AB is finite, just in the same sense the said time interval is finite – in answering this Aristotle emphasizes strongly the absolute congruence of the structure of the continuum in both its *extensive* and *durational* forms.

But a major difference between these two forms comes into view as soon as we observe that parts of the spatial continuum (and of what occupies space *continuously*, i.e. matter, for all ancient philosophy except the Atomists) *coexist* – whereas the parts of the temporal continuum (and of that which occurs in it, namely movement) *cannot coexist*. In fact Aristotle himself had raised serious doubts about the existence and reality of time in *Physics* Δ, 10, *and left them unanswered*, unlike what he did with the corresponding ἀπορίαι as to the existence of τόπος. This fact is duly emphasized by Simplicius [14], and before him by Damascius [15], possibly by Iamblichus as the ultimate source [16].

The main gist and point of Aristotle's [17] ἀπορίαι about the existence of time in the beginning of Δ, 10 is this: the past does not exist anymore; the future does not exist yet; the present is not a part of time but a limit (the "now" has strictly no temporal duration); if that of which a limit is the limit does not exist, then nor can the limit really exist; the present is the limit of past and future; from which propositions there follows necessarily that neither the past, nor the future nor the present exist – and hence that time does not exist at all.

The second aporematic course (from 218a8 sqq.) relates to the status of the νῦν (which is the only thing in time which can exist properly speaking if anything can) showing that it can neither be the same throughout the flow of time, nor different each moment.

This latter development can be met – after all everything in this World lies in the same predicament: it continually γίγνεται καὶ φθείρεται, yet it does have some δάνειον stability and identity coming down from its eternally immutably archetype. The really formidable objection is the former. How can it be answered? This is the first requisite for any adequate theory of Time.

A second prerequisite comes into light through Damascius' penetrating discernment of a fundamental disanalogy between extension and duration which

turns the tables against Aristotle by confronting him in his own terrain. In the former field there is no real objection to have a through and through continuity, since all the continuously interpenetrating (as it were) parts are *co-existent* [18]. And so the divisibility of magnitude and extended material substance does not undermine their cohesion. Magnitudes and physical entities are really broken, one may maintain with plausibility, when they are actually divided, and not on account of their inherent and essential divisibility. And divisibility as pure potentiality cannot threaten the integrity of magnitudes and material bodies. But in duration we have to do essentially with *passing* from one stage to another *whereby the former is extinguished and the latter comes to exist*; so we have a *real* and *actual* division at *each and every moment*; the division is effected *a parte rei* at each successive moment; so we cannot escape by invoking the distinction between divisibility as mere potentiality to be divided and actual division; in a certain sense if time is indefinitely divisible, then it must be *actually* divided into an infinity of elements – and this must happen every moment [19]. But then how can time proceed from any moment to any other? How can it do that if this would necessitate the execution of an *infinite* number of steps *in actuality*? Or indeed how could even an infinite number (of whatever order!) of momentary νῦν constitute ever a single step onwards? You have to actually take this step – if time is to move – - nothing similar happens with spatial extension; co-existence there solves the problem – or rather no problem is posited in that case at all.

Mutatis mutandis, the same holds for movement of any sort. Therefore time and movement must proceed *stepwisely*, καθ᾽ ἅλματα if they are to proceed at all [20].

But on the other hand χρόνος (and κίνησις) are συνεχῆ as well. Any time interval is divis*ible* indefinitely, must be so because of its very essence – there is no escape from this fundamental intuition. This is why Damascius calls the time *συνεχές and διωρισμένον* μέγεθος – a "discrete continuum", as distinguished from the extensional μέγεθος, the ἀληθῶς συνεχές. But how are these contradictory characteristics to be combined?

Here Damascius' originality comes especially to the fore. The Xenocratean tendency was to make a συνεχές [21] out of ἀμερῆ; Damascius makes it out of units which are *συνεχεῖς* within themselves as it were, but *consecutive,* διωρισμένα, the one upon and from the other! This ingenious theory is hinted in a few words in Dubitationes et Solutiones in Platonis Parmenidem, ed. Ruelle, §389, vol. II, p. *236.8-19*. And these units are the μέτρα, the ultimate measures of χρόνος. Such a unit is each νῦν. Here is the locus classicus for the gist of the theory: εὐλαβητέον ἄρα τὸ ἐξ

ἀμερῶν εἶναι τὸν χρόνον. τί οὖν; οὐδὲ συνεχής ἐστι καὶ διωρισμένος, ὡς ἀποδείκνυσιν (sc. Proclus); πάνυ γε φήσω, ἀλλ' οὐκ ἐκ μερῶν ἀμερῶν, ἀλλ' ἐκ διαστατῶν διωρισμένων συγκείμενος. ἔστιν γὰρ σύνθετος, ὥς φησι Στράτων [Strato elaborated on the arguments against the existence of time precisely for this reason, cf. Simplicius, Corollarium de Tempore, ad fin., p. 800.17 Diels], ἐκ μερῶν μὴ μενόντων (and so the elemental parts must be discrete so that the present may be saved from sinking into the nonexistence of past and future, as I argued above): ταύτῃ οὖν ἐκ διωρισμένων. ἕκαστον δὲ μέρος συνεχές ἐστι, καὶ οἷον μέτρον ἐστὶν ἐκ πολλῶν μέτρων. ἐδείκνυμεν γὰρ καὶ ἐν τοῖς εἰς Τίμαιον ὅτι οὐ κατὰ τὰ νῦν προκόπτει ὁ χρόνος – οὐδὲ γὰρ ἄν προέκοψεν ἀπείρων ὄντων ἀεὶ τῶν νῦν. (that is time would not flow if its flow was realized through a postulated succession of dimensionless moments, according to the reasoning explicated supra). ἀλλ' ὥσπερ ἡ κίνησις προκόπτει διαστηματικῶς (i.e. stepwisely), ἀλλ' οὐ κατὰ σημεῖον, ἀλλ' οἷον καθ' ἅλματα (i.e. by ultimately elemental jumps), ὡς ἔλεγεν ὁ Ἀριστοτέλης, οὕτως ἀνάγκη καὶ τὸν χρόνον κατὰ μέτρα ὅλα προβαίνειν ἅ μετρητικά τῶν ἁλμάτων γίγνεται τῆς κινήσεως. οὕτως ἄρα ἐκ μέτρων ὁ χρόνος, ἀλλὰ μέτρων πέρασι διειλημμένων καὶ διωρισμένων.

This theory meets then perfectly the second requirement, i.e. that we must account for the possibility of actually *passing* from one stage to another (as the existent present passes into the nonexistence of the past and makes place for the nonexistent proximate future coming into existence as present), and therefore of *actual* division at each step. It also solves our difficulties about the existence of time in general – in fact, according to Damascius, it is the only way of meeting the first requirement above mentioned. The present is an interval of time in fact, no mere limit – and it exists *all at once*, tangibly, as it were, thereby illustrating its descent from eternity which is the source of all time. And this extended νῦν [22] is the position of time which exists fully at each present moment [23].

Before settling the last important question as to how exactly is Damascius conceiving of these divisible units or *divisible (from a mere logicalpoint of view) but undivided and metaphysically indivisible* units of time and movement, it is highly important to observe that Damascius thinks that in his solution he is in agreement with Aristotle. See 236.15-16 above quoted and apud *Simplicium, Corollarium de Tempore, from his important Commentaries on Aristotle's Physics, p.* 796.32-797.13 Diels. Damascius had probably in mind chapter 10 of Book Z, Physics, just after the discussion of Zeno's arguments against κίνησις. See esp. Physica, 241a6-26, in particular such expressions as 241a6-7 or 15: ἔτι δὲ καὶ ἐκ τῶνδε φανερὸν ὅτι οὔτε

στιγμὴν οὔτ' ἄλλο ἀδιαίρετον οὐθὲν ἐνδέχεται κινεῖσθαι and ἔτι δ' εἰ ἅπαν ἐν χρόνῳ κινεῖται, *ἐν δὲ τῷ νῦν μηθέν* etc. This is the nearest I think to his view. But if so, he chose not to notice that Aristotle draws there from the impossibility of *passing* and *movement* in a moment, in a νῦν as πέρας, just the opposite of what Damascius is inferring, namely that change and passing from one stage to another is συνεχές (cf. 235b24-25), not that it is executed stepwisely in a discrete field. Obviously Damascius thought that it is impossible for Aristotle to blunder so badly [24], that he ought to have meant what Damascius says [25]. Of course Simplicius correctly and conscientiously detects the pious fraud – see 797.26 sqq.

And now I come to the final question proposed above. What about those *elemental* units and ultimate *measures* of time which are the core of Damascius' singular theory. We have seen that they are divisible as such (being a continuum) and yet ultimate undivided (and metaphysically indivisible) atoms. How are we to reconcile and combine these characteristics? Damascius' answer is briefly indicated [26] in 242.9-14. And if I am right, it is precisely in the Platonic spirit of my development in pp. 7-8 above. Ταῦτα δ' οὖν τὰ ἅλματα μέτρα ὄντα χρονικὰ δημιουργικαῖς τομαῖς διωρισμένα *καὶ ταύτη γε ἀμέριστα*, καὶ ὅλον ὁμοῦ ἕκαστον τὴν ἐπίσχεσιν τοῦ πορευομένου χρόνου φατέον ἐνδείκνυσθαι, καὶ ὧν καλεῖσθαι, οὐχ ὡς πέρας χρόνου, ἀλλ' ὡς χρόνον ἀμέριστον δημιουργικῶς, εἰ καὶ τῇ ἡμετέρᾳ ἐπινοίᾳ διαιρετόν [what I described above as merely logically, in sheer intellectual conceptualization, divisible, but undivided and *metaphysically indivisible*], καὶ τοῦτο ἐπ' ἄπειρον, ἐπεὶ καὶ πᾶν σῶμα ἐπ' ἄπειρον διαιρετόν, ἀλλ' εἰσὶν ἀμέριστοι δημιουργικαὶ τομαὶ τῶν σωμάτων, etc. This is the crucial point. The δημιουργία of this world implies the injection of order and stability about which I have spoken. At the fundamental level this injection consists in the imposition of an indivisibility (in imitation of the παντελὴς ἀμεριστότης of the higher realities) upon what is in itself divisible indefinitely. This is part, a fundamental part, of the imposition of order upon the initial Disorder. So far as the World exists (that is, for ever), as long as the demiourgic activity of the higher reality is exercised on matter, these elementary units of time cannot be broken down – they "were" divisible in themselves, but now they are divisible only in our thought in itself as dissociated from ontological reality. God has seized upon them, and his inflexible law strengthens them into unbreakability: it is only thus that orderly development, indeed development at all, is made possible, as Damascius goes on to explain in more detail.

Again, there is much flesh that can be added to this skeletal structure so that the full beauty of the theoretical form may be rendered manifest. As for some secondary confirmation, notice p.242.14-15, where Damascius claims the same theory for spatial and bodily extension, something which is not necessitated by the above mentioned essential differing characteristic of duration vis-à-vis extension, *but which obviously comes fresh from an interpretation of Timaeus*, in the spirit of what I expounded above. The demiurgic "cuts" create and set off the elemental units of extension and duration imposing necessary form into the corresponding continua, and thus ennabling something fundamentally incapable of beingness (the continuum) to exist as an ordered field under a defined metric.

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As for Simplicius, (whose both Corollaries, on space and time, in his valuable Commentaries on the Aristotelian Physics, are admirable), he cannot agree with the Damascian peculiarities and idiosyncrasies – as he is viewing them. *He remains a most orthodox Aristotelian*. See for instance p.775.3-12 Diels. But above all see his concluding remarks, from p.798.9 onwards to the end. It is a first-class blunder of Diels, that he ascribes all these final pages from 797.36 onwards to Damascius (ending with 800.16). In fact the Damascian verbatim quotation, probably from his special treatise Περὶ χρόνου, ends with 798.9. What follows is inconsistent with the Damascian positions as explained by Simplicius previously and as contained in the *In Parmenidem* passages; Simplicius in fact justifies the purely Aristotelian standpoint of an absolute *continuity* of all extensional and durational μεγέθη [27]. Besides it would be awkward, indeed absurd, to say what Simplicius says in 800.19-21 after he had given a long quotation probably from that same book Περὶ χρόνου, referred in this passage, with the account of the solution of the Aristotelian ἀποριαι as to the existence of time.

It is important to notice Simplicius' viewpoint: 798.26-799.10. The emphasis lies in the mode of thinking essentially belonging to the psychic substance. But the absolute flow of time and becoming is emphasized in all its Aristotelian implications regarding their continuity.

NOTES

[1] The expression is Simplicius', Commentaries in *Physics*, 142.16 sqq. (Diels) (= Xenocrates Fr. 47 Heinze), where he unsuccessfully tries to apply it to Xenocrates' position. If I am right in distinguishing (a) from (b) in the way I do, it is clear that

Simplicius' defense is inapplicable to a Xenocratean type of doctrine, but could be promoted by an Atomist, when appropriately modified.

All the Neoplatonists (pace Damascius!) took for granted the *Aristotelian position* of the matter (in this as in so many other topics). It is very instructive to observe the various types of rescue operation undertaken by them on behalf of Xenocrates – always on an Aristotelian understanding of the whole question. See Porphyrius apud Simplicius *In Phys*. 140.6 sqq. Diels (= Xenocrates Fr. 45), Proclus, *In Timaeum*, p. 215e (= Xenocrates Fr. 46) and Syrianus, *In Metaphysica*, 902b18 Uscener (= Xenocrates Fr. 46). Best of all, Xenox Xenocrates (Heinze) pp. 173-178 – gives absolutely *all* the important passages on the matter.

[2] σώματα may be mathematical entities, without matter, a mathematical cube for instance, or physical, complete bodies. Some of the ancients did not pay much regard to this distinction between extensionality and corporeality, it would seem, but most of them expressly utilized it.

[3] Notice Aristotle's expression about Xenocrates (again without mentioning him), that his theories treat of μαθηματικῶν οὐ μαθηματικῶς (v. *Metaphysica* 1080b28).

[4] There is absolutely nothing in the Περὶ ἀτόμων γραμμῶν which could not come from Aristotle. Indeed till we see some powerful exposition of reasons for doubting its authenticity, I will consider it as by Aristotle.

[5] Already this seems to invite further comment. If I am right Damascius may be said to deny, in a sense, this very assertion. See below.

[6] Cf. e.g. *Physica*, 231a24.

[7] Aristotle distinguishes habitually (and here, too) that which is in *continuation* of, and in continuity with another (συνεχές) from that which is in *touch* with another (ἅπτεται), and both from that which is numerically and separately *consecutive* or *after* another (ἐφεξῆς) – and the argument in (2) is pursued as to both the members of the first division, and one can see that the complication is really irrelevant as to the structure of the argument here.

[8] And even worse! He says (231a28): οὐ γάρ ἐστιν ἔσχατον τοῦ ἀμεροῦς οὐδέν· ἕτερον γὰρ τὸ ἔσχατον καὶ οὗ ἔσχατον. Right as to the second phrase; but he himself usually maintains that the πέρας is not a *μέρος* of that of which it is a πέρας. So nothing hinders an ἀμερές to have a πέρας – unless you project the commonsensical notion of line onto the ἄτομος line. What is really at work here is that we cannot *imagine* any magnitude, however small, which has limits and which is not further divisible; but then inability to imagine is not due to any selfcontradiction

or inconsistency in the idea of its having limits, but to the strong commonsensical intuition bare and blunt and blind we have that whatever is extended is indefinitely divisible – that's the end of it!

[9] In Modern Mathematics, for many centuries such a useful figment was the notion of infinitesimal (ἀπειροστόν) – which in the past century was discarded and substituted by more scientific and correct conceptions. But it served in the development of mathematics *just as well as, if not better than, the correcter conceptions* might have helped!

[10] An interesting testimony is *Plutarch's* treatment of the question why Plato did not give an elementary shape as the element of all περιφερῆ σχήματα καὶ κυκλικά, just as he did for the εὐθύγραμμα, while he admitted the fundamental division of all lines into εὐθεῖαι and περιφερεῖς (Ζήτημα Ε΄, in *Platonicae Quaestiones*). He argues in effect (see §§2, 3 and, especially, 4) that the περιφερεῖς γραμμαί (like the circumference of a circle) are made up of small εὐθύγραμμαι (ἄτομοι), and therefore need no special principle for them. [The reference I owe to Robin, who in turn owes it to Apelt].

[11] 53c-55c.

[12] As to Xenocrates, he maintained that the mathematical are *identical* with the *ideal* numbers and magnitudes. It is not surprising therefore (it is indeed the peculiarity of the theory) to find everywhere in his views that tension between the purely mathematical and the metaphysical viewpoints which made Aristotle say that he spoke of μαθηματικά οὐ μαθηματικῶς. In a certain sense Xenocrates goes back to primitive Pythagoreanism – but with all the subsequent elaborate techniques. It is an extremely important phenomenon in the History of Ideas in general: a type of degeneracy perhaps, whereby the entire developed apparatus of a sophisticated age is brought to bear and to elaborately support a more or less "archaic" position. The most illustrious example in the domain of Ancient Thought is Iamblichus. One should carefully distinguish this type of view from that other which *reinterprets*, and therefore basically "explains away", the old Idea in terms of the new, sophisticated consciousness. No, according to the standpoint I mean, the whole of the highly perfected nexus of methods and understanding of the later age is simply *used* as a *means*, as a weapon and implement to re-establish the perhaps antiquated Idea in its very "archaic" peculiarity – not as helping to bring out the analogy or correspondence of that Idea with its modern "equivalent".

[13] If I am roughly right in the above solution to the problem, then one also sees the core of truth in the Neoplatonic rescue operations signaled above (n. [1]).

[14] V. *Corollarium de Tempore*, p.795.27 sqq. Diels. Simplicius adds that no commentator supplied the want by providing the λύσεις of the relevant ἀπορίαι.

[15] V. *Corollarium de Tempore*, p.796.26 – cf. *Damascius, in Parmenidem,* §390. This is the point of the twelfth ἀπορία there.

[16] This I infer from the fact that Damascius' solution is given by Iamblichus according to Simplicius *op.cit.* p.793.22-23. But since Simplicius seems there just to recapitulate what Iamblichus said in the immediately preceding quotation, and since I also think that there Iamblichus meant to apply his remarks about the ἀμερὲς νῦν not to a portion of present time but to the transcendent principle of the immanent νῦν which, running through the time as an unbroken thread, maintains the order of time κατὰ τὸ πρότερον καὶ ὕστερον (cf. the developments in Simplicius' commentary on *Categoriae*) – for these reasons I am inclined to think that Iamblichus' point was different; which, on the other hand, does not exclude the possibility of Damascius' being inspired by the corresponding Iamblichean doctrine – though I would connect with that the other (no doubt again related) Damascian doctrine about which Simplicius feels so much opposed, v. *op.cit.* 775.31-34, as well as the whole related subsequent development of Simplicius' objections to the peculiar Damascian view that time exists simultaneously also all together, τὸ εἶναι ἅμα τὸν ὅλον χρόνον ἐν ὑποστάσει – as if not ἐν διεξόδῳ. (This aspect, by the way, will be extremely useful to any serious treatment of the question of αἰών and χρόνος in Aristotle. See the Damascian quotation 780.20 sqq., esp. 780.33-781.13, and the following objections of Simplicius.

[17] But notice that inquieting καὶ διὰ τῶν ἐξωτερικῶν λόγων in *Physica*, 217b31.

[18] V. Damascius, In *Parmenidem,* p.236.24-25 Ruelle – διορισμός here is a technical term referring to the *διωρισμένον μέγεθος* as opposed to συνεχές, discrete system as against continuum. We have a διωρισμένον μέγεθος where there is an ἐφεξῆς as Aristotle says, or a sequence of parts in contact one to another, where between two consecutive elements there is no element of the same kind.

[19] This is the deep sense of *op.cit*. p.236.11-12. Strato was ingenious, but perverted in the Peripatetic way! See his theory of time, apud Simplicius, p.788.36 sqq.

[20] See p.236.13-16. And see esp. p.236.21-25 where the whole nexus is succinctly put. "συνεχίζεται" means is made συνεχές; this is done by the spatial extension of the body moving, and of the transversed distance; "διορίζεται" means is made διωρισμένον μέγεθος as well; and this is done διακοπτομένη (sc. the

κίνησις); for there is disappearance of the previous state and appearance of the new, hence division a parte rei, whether we wish it or not, whether we consider it then or not.

[21] For to this extent, I do not think that Aristotle willfully misrepresented his adversaries when he objected to the view that a συνεχές cannot be made out of ἀμερῆ. This thesis was probably their view, they would not rather deny that μεγέθη (apart from the consecutive ἄτομα) *are* συνεχῆ. But having written this, I am not so sure of it. In any case, however, the statement in the text holds good, for irrespective of what, say, Xenocrates has thought, in effect his view was bound to be taken in this way, for nobody could in good conscience suffer to alienate himself from the fundamental intuition that extension and duration are indeed συνεχῆ.

[22] To be distinguished from the ἐξαίφνης of the 3rd Parmenidean hypothesis; the extended νῦν is, for Damascius, the νῦν occurring at the end of the 2nd hypothesis.

[23] There is another Damascian doctrine which is calculated also to meet a more fundamental *ἀπορία* as regards the existence of time. For we cannot after all help asking: do really the past and the future not exist? Not at all? Is the existential status of something that did happen in the past exactly the same with something which did *not* happen, or even with something which might have happened but was prevented from happening, might had occurred but just did not happen to occur? Is it not the whole time a "well-rounded whole" (to speak in Parmenidean fashion) like the whole extension? Does it not form a certain unity, and therefore a kind of existence, in the divine eyes? Such movement of thought lies perhaps at the bottom(*) of that other peculiar Damascian view, to which Simplicius was so opposed, and according to which Time has, at a higher level, a *simultaneous* existence as a hypostatic whole – an existence lying between αἰών, eternity, and χρόνος ἐν διεξόδῳ and part-after-part-succession. Cf. n. [16].

(*) There are also, of course, more "rationalistic" reasons for it, as detailed by Simplicius!

[24] For in the name of the Supreme, what else can logically you be saying if you deny that κίνησις is effected in a moment than that it requires a unit of time for a unit of movement to be realized? By the beauty of Apollo, what else does it mean to say that the κίνησις takes place συνεχῶς and not δι᾽ ἁλμάτων, than to claim that it must happen in νῦν after νῦν and thus to directly contradict the former position? Is it not here at work the phantasm of the infinitisernals? *But Aristotle's attitude in such matters is wonderfully and frankly confessed in a most significant passage – Περὶ ἀτόμων*

γραμμῶν 969b3-6!!!: ἀλλ' ἄτοπον ἴσως τὸ μὴ δυναμένους λύειν τὸν λόγον δουλεύειν τῇ ἀσθενείᾳ, καὶ προσεξαπατᾶν ἑαυτοὺς μείζους ἀπάτας, βοηθοῦντας τῇ ἀδυναμίᾳ. Exactly what Aristotle was thinking of those that succumb to the Eleatic reasonings, from which I started this inquiry.

[25] For a clear cut instance of such an attitude v. *Damascius* apud Simplicium, *Corollarium de Tempore,* p.780.20-781.13. For the "sane", Aristotelian-like answer by Simplicius see what follows upon that passage. But Damascius could not see any other alternative than the two implied in n. [24] above – see *In Parmenidem,* p.242.5-6.

[26] The full analysis was probably given in his *Commentary on Timaeus*, and in his treatise on *Χρόνος*.

[26] He maintains the exactly opposite to the Damascian thesis, formulated in almost the Damascian terms: 798.23-26 (where read ἀπώλεσας or ἀπολέσει (with a) instead of the unsyntactical ἀπολέσας).